SHIVAJI UNIVERSITY, KOLHAPUR

****

Accredited By NAAC with 'A++' Grade

Online Mode

Syllabus For

M.Sc. Mathematics Part-I

Semester I And II

(Syllabus to be Implemented from June,2021 onwards.)

Title of the course : M.Sc .(Mathematics) (Online Mode)

1) M.Sc.(Mathematics) Programme has semester pattern

2) Duration of the Program

- The M.Sc. programme will be a full-time two years **online mode** i.e. 4 semesters. Pattern of examination will be Semester System.

Medium of Instruction

- The medium of Instruction will be English only.

Admission Procedure

- Eligibility: ,B.Sc.Mathematics,
- Admission through University Entrance exam only..
- Reservation of Seats as per rules of Government of Maharashtra.

3) Structure of the course

The following table gives the scheme of Examination at M.Sc. Mathematics (Part I) according tothe New Syllabus and pattern of Examination.

M.Sc.(Mathematics) Semester - I

Course Code	Title of course	Number ofCredits	Duration of Semester endExam in hrs.	Marks-Term end Exam	Marks-(Internal) Mid Semester Exam	Total marks of Each Subject
OMT 101	Advanced Calculus	5	3	90	30	120
OMT 102	Linear Algebra	5	3	90	30	120
OMT 103	Complex Analysis	5	3	90	30	120
OMT 104	Classical Mechanics	5	3	90	30	120
OMT 105	Ordinary Differential Equations	5	3	90	30	120

M.Sc.(Mathematics) Semester - II

Course Code	Title of course	Number of Credits	Duration of Semester end Exam in hrs.	Marks-Term end Exam	Marks-(Internal) Mid Semester Exam	Total marks of Each Subject
OMT 201	Functional Analysis	5	3	90	30	120
OMT 202	Algebra	5	3	90	30	120
OMT 203	General Topology	5	3	90	30	120
OMT 204	Numerical Analysis	5	3	90	30	120
OMT 205	Partial Differential Equations	5	3	90	30	120

Syllabi of M.Sc. Mathematics (Part I) (Online Mode)

M. Sc. (Mathematics) (Part I) (Semester I)

(Online Mode)

(Introduced from June 2021 onwards)

Course Code: OMT 101

Title of Course: Advanced Calculus

Course Outcomes :Upon successful completion of this course, the student will be able to:

(i) Analyze convergence of sequences and series of functions
(ii) check differentiability of functions of several variables
(iii) Apply inverse and implicit function theorems for functions of several variables
(iv) Use Green's theorem, Stoke's Theorem, Gauss divergence Theorem.

Unit 1 : [15 Lectures]

Sequences and series of functions: Pointwise convergence of sequences of functions, Examples of sequences of real valued functions, Definition of uniform convergence, Uniform convergence and continuity, Cauchy condition for uniform convergence, Uniform convergence and Riemann integration, [1,2]

Unit 2: [15 Lectures]

Uniform convergence and differentiation, Equicontinuous family of functions.Multivariable differential Calculus: The Directional derivatives, directional derivatives and continuity, total derivative, total derivatives expressed in terms of partial derivatives, The matrix of linear function, mean value theorem for differentiable functions, [2,1]

Unit 3: [15 Lectures]

A sufficient condition for differentiability, sufficient condition for equality of mixed partial derivatives, Taylor's formula for functions from R^n to R^1.Implicit functions: Functions of several variables, [1]

Unit 4: [15 Lectures]

Linear transformations, Differentiation, Contraction principle, The inverse function theorem, The implicit function theorem and their applications.Integral Calculus: Path and line integrals, Multiple integrals Double integral (Theorems without proof) Application to area and volume.(Theorems without proof)

Unit 5: [15 Lectures]

Greens theorem in the plane. Application of Green's Theorem. Change of variables, special cases of transformation formula. Surface integral, change of parametric representation.Other notations for surface integrals, Stoke's Theorem Curl and divergence of a Vector field.Gauss divergence Theorem. [3]

Recommended books :

1) Principles of mathematical Analysis, Walter Rudin, third Edition, McGraw Hill book company
2) Mathematical Analysis, Apostal, Second Edition, Narosa Publishing House.

3) Calculus Vol. II , Tom M. Apostol, Second EditionWiley India Pvt. Ltd.

Reference books:

1) W.Fleming, Functions of several Variables,2nd Edition ,Springer Verlag, 1977
2)J.R.Munkres, Analysis on Manifolds.

M. Sc. (Mathematics) (Part I) (Semester I)
(Online Mode)
(Introduced from June 2021 onwards)

Course Code : OMT 102
Title of Course: Linear Algebra

Course Outcomes: Upon successful completion of this course, the student will be able to:
1. understand basic notions in Linear Algebra and use the results in developing advanced mathematics.
2. study the properties of Vector Spaces, Linear Transformations, Algebra of Linear Transformations and Inner product space in some details.
3. construct Canonical forms and Bilinear forms.
4. apply knowledge of Vector space, Linear Transformations, Canonical Forms and Bilinear Transformations.

Unit I: **[15 Lectures]**
Basic concepts of vector space, Dual Spaces, Annihilator of a subspace, Quotient Spaces. Inner product spaces,.

Unit II: **[15 Lectures]**
Algebra of Linear transformationsEigen values and eigenvectors of a linear transformation. Diagonalization..

Unit III: **[15 Lectures]**
Invariant subspaces, Similarity of linear transformations,Triangular form, Nilpotent transformations, Primary decomposition theorem, Jordan blocks and Jordan forms,.

Unit IV: **[15 Lectures]**
Rational Canonical Form, Trace and transpose, Determinants, Real Quadratic forms,Hermitian, Self adjoint, Unitary and normal linear transformation.

Unit V: **[15 Lectures]**
Symmetric bilinear forms, skew symmetric bilinear forms, Group preserving bilinear forms.

Recommended Book(s):

1. Herstein I. N. : Topics in Algebra, 2nd Edition, Willey Eastern Limited.
2. Hoffman, Kenneth and Kunze R: Linear Algebra, Prentice Hill of India Private Limited., 1984.

Reference Books:

1. A. R. Rao and P. Bhimashankaran, Linear Algebra, Hidustan Book Agency.
2. Surjit Singh, Linear Algebra, Vikas publishing House (1997).
3. Gilbert Strang: Introduction to Linear Algebra, Wellesley-Cambridge Press

M. Sc. (Mathematics) (Part I) (Semester I)
(Online Distance Mode)
(Introduced from June 2021 onwards)

Course Code : OMT 103

Title of Course: Complex Analysis

Course Outcomes: Upon successful completion of this course, the student will be able to:

1. understand fundamental concepts of complex analysis.
2. identify analytic functions, Conformal maps.
3. construct Taylor and Laurent series.
4. classify singularity and apply Residue Theorem to evaluate real integrals.
5. enjoy the beauty of analytic functions and related concepts.

Unit 1: **[15 Lectures]**

Power series, Radius of convergence, analytic functions, Cauchy-Riemann equations, Harmonic functions, Conformal mappings, Mobius Transformations.

Unit 2: **[15 Lectures]**

line integral,Power series representation of analytic functions, zeros of an analytic function, Liouville's Theorem, Fundamental theorem of algebra, maximum modulus theorem, the index of a closed curve.

Unit 3: **[15 Lectures]**

Cauchy's theorem and integral formula, Morera's Theorem,Counting zeros, open Mapping theorem, Goursat's Theorem, classification of singularities, Laurent series development.

Unit 4: **[15 Lectures]**

Casorati–Weierstrass theorem, residues, residue theorem, evaluation of real integrals,The argument principle, Rouche's theorem, the maximum principle.

Unit 5: **[15 Lectures]**

Schwarz's lemma and its application to characterize conformal maps, Riemann mapping theorem.

Recommended Book :

1. J. B. Conway: Functions of One Complex Variable (3rd Edition) Narosa Publishing House.

References :

1. S.Ponnusamy, Foundations of Complex Analysis, Narosa Publishing House.
2. Alfors L. V.: Complex Analysis, McGraw 1979.
3. Churchill and Brown, Complex Variables and applications, MacGrawHill(India). (8^{th} Edition, 2014)
4. Serge Lang, Complex Analysis, Springer
5. Steven G. Krantz, Complex Analysis, A Geometric view Point, The Carus Mathematical Monographs.
6. T. W. Gamelin, Complex Analysis, Springer.

M. Sc. (Mathematics) (Part I) (Semester I)
(Online Mode)
(Introduced from June 2021 onwards)

Course Code : OMT 104

Title of Course: Classical Mechanics

Course Outcomes: Upon successful completion of this course, the student will be able to:

1. discuss the motion of system of particles using Lagrangian and Hamiltonian approach.
2. solve extremization problems using variational calculus.
3. discuss the motion of rigid body.
4. construct Hamiltonian using Routh process.
5. use infinitesimal and finite rotations to analyze motion of rigid body.

UNIT – I: **[15 Lectures]**

Mechanics of a particle, Mechanics of a system of particles, conservation theorems, constraints, Generalized coordinates, D' Alembert's Principle, Lagrange's equations of motion, Simple applications of Lagrangian formulation, Cyclic co-ordinates and generalised momentum.

UNIT – II: **[15 Lectures]**

conservation theorems,Functionals, basic lemma in calculus of variations, Euler- Lagrange's equations, first integrals of Euler- Lagrange's equations, Geodesics in a plane and space, the minimum surface of revolution, the case of several dependent variables Undetermined end conditions. the problem of Brachistochrone, Isoperimetric problems, problem of maximum enclosed area.

UNIT – III: **[15 Lectures]**

Hamilton's Principle, Derivation of Hamilton's principle from D'Alembert's principle, Lagrange's equations from Hamilton's principle. Lagrange's equations of motion for nonconservative systems (Method of Langrange's undetermined multipliers), Hamiltonian function, Hamilton's canonical equations of motion,

UNIT – IV: **[15 Lectures]**

cyclic co- ordinates and Routh's procedure, Derivation of Hamilton's equations from variational principle, Physical significance of Hamiltonian, The principle of least action.Orthogonal transformations, Properties of transformation matrix, infinitesimal rotations.The Kinematics of rigid body motion: The independent co-ordinates of a rigid body, the Eulerian angles,

Unit V: **[15 Lectures]**

Euler's theorem on motion of rigid body, Angular momentum and kinetic energy of a rigid body with one point fixed, the inertia tensor and moment of inertia, Euler's equations of motion, Cayley- Klein parameters, Matrix of transformation in Cayley- Klein parameters, Relations between Eulerian angles and Cayley- Klein parameters.

Recommended Books :

1) Goldstein, H. Classical Mechanics. (1980), Narosa Publishing House, New Delhi.
2) Weinstock: Calculus of Variations with Applications to Physics and Engineering (International Series in Pure and Applied Mathematics). (1952), Mc Graw Hill Book Company, New York.

Reference Books :-

1) Whittaker, E. T. A Treatise on the Analytical Dynamics ofParticles and Rigid Bodies. (1965), Cambridge University Press.
2) Gupta, A. S. Calculus of Variations with Applications (1997), Prentice Hall of India.
3) Gelfand, I. M. and Fomin, S. V. Calculus of Variations (1963), Prentice Hall of India.
4) Rana, N.C. and Joag, P. S. Classical Mechanics. (1991) Tata McGraw Hill, New Delhi.

M. Sc. (Mathematics) (Part I) (Semester I)
(Online Mode)
(Introduced from June 2021 onwards)

Course Code : OMT 105

Title of Course: Ordinary Differential Equations

Course Outcomes: Upon successful completion of this course, the student will be able to:

1. study basic notions in Differential Equations and use the results in developing advanced mathematics.
2. solve problems modeled by linear differential equations
3. use power series methods to solve differential equations about ordinary points and regular singular points.
4. construct approximate solutions using method of successive approximation.
5. establish uniqueness of solutions.

Unit – I : [15 Lectures]

Linear Equations with variable coefficients: Initial value problems for the homogeneous equations. Solutions of the homogeneous equations, The Wronskian and linear independence, Reduction of the order of a homogeneous equation, The non-homogenous equations, Homogeneous equations with analytic coefficients.

Unit - II: [15 Lectures]

,**Linear Equations with regular singular points:** The Legendre equations ,The Euler equations, Second order equations with regular singular points The Bessel equation.

Unit - III: [15 Lectures]

, **Existence and uniqueness of solutions to first order equations:** Regular singular points at infinity ,The method of successive approximations, The Lipschitz condition of the successive approximation. Convergence of the successive approximation..

Unit – IV: [15 Lectures]

,**Existence and Uniqueness of Solutions to System of first order ordinary differential equations:** Non-local existence of solutions, Approximations to solutions and uniqueness of solutions An example- Central forces and planetary motion, Some special equations, Systems as vector equations.

Unit V: [15 Lectures]

Existence and uniqueness of solutions to systems: , Existence and uniqueness for linear systems, Green's function, Sturm Liouville theory.

Recommended books:

1) E.A.Coddington: An Introduction to Ordinary Differential Equations. (2012) Prentice Hall of India Pvt.Ltd. New Delhi.

2) G. Birkoff and G.G.Rota: Ordinary Differential Equations, John Willey and Sons

3) Mark Pinsky: Partial Differential Equations and Boundary-value Problems with Applications, AMS,3rd edition(2011).

Reference books:

1. G.F. Simmons Differential Equations with Applications and Historical note, MeGraw Hill, Inc. New York. (1972)
2. E.A. Coddington and Levinson: Theory of Ordinary Differential Equations, McGraw Hill, New York(1955)

3.E.D. Rainvills :Elementary Differential Equations,TheMacmillan company, New York. (1964)

M. Sc. (Mathematics) (Part I) (Semester II)
(Online Mode)
(Introduced from June 2021 onwards)

Course Code: OMT 201

Title of Course: Functional Analysis

Course Outcomes: Upon successful completion of this course, the student will be able to:

1. understand the fundamental topics, principles and methods of functional analysis.
2. demonstrate the knowledge of normed spaces, Banach spaces, Hilbert space.
3. define continuous linear transformations between linear spaces, bounded linear functionals.
4. apply finite dimensional spectral theorem.
5. identify normal, self adjoint, unitary, Hermit ion operators.

Unit I: **[15 Lectures]**

Normed linear spaces, Banach spaces, Quotient spaces, Continuous linear transformations, Equivalent norms, Finite dimensional normed spaces and properties, Conjugate space and separability.

Unit II: **[15 Lectures]**

The Hahn-Banach theorem and its consequences,Second conjugate space, the natural embedding of the normed linear space in its second conjugate space, Reflexivity of normed spaces, The open mapping theorem, Projection on Banach space.

Unit III: **[15 Lectures]**

the closed graph theorem, the conjugate of an operator, the uniform boundedness principle,Hilbert spaces: examples and elementary properties, Orthogonal complements, The projection theorem, Orthogonal sets.

Unit IV: **[15 Lectures]**

The Bessel's inequality, Fourier expansion and Perseval's equation, separable Hilbert spaces, The conjugate of Hilbert space, Riesz's theorem, The adjoint of an operator,Self adjoint operators, Normal and Unitary operators.

Unit V: **[15 Lectures]**

Projections, Eigen values and eigenvectors of an operator on a Hilbert space, The determinants and spectrum of an operator, The spectral theorem on a finite dimensional Hilbert space.

Recommended Book(s):

1. G. F. Simmons, Introduction to Topology and Modern Analysis, Tata McGraw Hill, 1963.

Reference Books:

1. Erwin Kreyszig, Introductory Functional Analysis with Applications, John Wiley and Sons, 1978
2. A. E. Taylor, Introduction to Functional analysis, John Wiley and sons,1958.
3. J. B. Convey, A Course in Functional Analysis, Springer-Verlag, 1985.
4. G. Bachman and L. Narici, Functional Analysis, Academic Press, 1972.
5. B. V. Limaye, Functioned Analysis, New age international, 1996.

M. Sc. (Mathematics) (Part I) (Semester II)
(Online Distance Mode)
(Introduced from June 2021 onwards)

Course Code: OMT 202

Title of Course: Algebra

Course Outcomes: Upon successful completion of this course, the student will be able to:

1. study group theory and ring theory in some details.
2. introduce and discuss module structure over a ring.
3. apply Sylow theorems.
4. use homomorphism and isomorphism theorems.
5. check irreducibility of polynomials over Q using Eisenstein criteria.

Unit I: **[15 Lectures]**

Groups of permutations, Simple groups, simplicity of An (n > 5), Commutator subgroups, normal and subnormal series, Jordan-Holder theorem, Solvable groups, isomorphism theorems.

Unit II: **[15 Lectures]**

Zassenhaus Lemma, Schreier refinement theorem,Group action on a set, fixed sets and isotropy subgroups, Burnside theorem, Sylow theorems, p-groups.

Unit III: **[15 Lectures]**

Applications of the Sylow theory and Class equation,Rings of polynomials, factorization of polynomials over fields, the division algorithm in F[x], irreducible polynomials.

Unit IV: **[15 Lectures]**

Eisenstein criteria, ideals in F[x], uniqueness of factorization in F[x], unique factorization domains, principal ideal domain, Gauss lemma, Euclidean DomainsModules, sub-modules, quotient modules.

Unit V: **[15 Lectures]**

homomorphism and isomorphism theorems,fundamental theorem for modules, Simple modules, Schur's lemma, Artinian and Noetherian modules.

Recommended Book(s):

1. John B. Fraleigh , A first course in Abstract Algebra (Third Edition), Narosa publishing house, New Delhi.
2. C. Musili, Introduction to Rings and Modules (Second Revised Edition), Narosa Publishing house, New Delhi.

Reference Books:

1. Joseph A. Gallian, Contemporary Abstract Algebra (Fourth Edition), Narosa Publishing house, New Delhi.
2. Bhattacharya, Jain and Nagpal, Basic Abstract Algebra, 2ndedition, Narosa Publishing House, New Delhi.
3. I. N. Herstein, Topics in Algebra, Vikas Publishing House.
4. N. Jacobson, Basic Algebra, Hind Publishing Corporation, 1984.

M. Sc. (Mathematics) (Part I) (Semester II)
(Online Mode)
(Introduced from June 2021 onwards)

Course Code: OMT 203

Title of Course: **General Topology**

Course Outcomes: Upon successful completion of this course, the student will be able to:

1. built foundations for future study in analysis, in geometry, and in algebraic topology.
2. introduce the fundamental concepts in topological spaces.
3. acquire demonstrable knowledge of topological spaces, product spaces, and continuous functions on topological spaces.
4. identify compact and connected sets in topological spaces.
5. use Separation and countability axioms, Urysohn lemma, Urysohn metrization theorem and the Tychonoff theorem.

Unit I: **[15 Lectures]**

Topological Spaces, Basis for a Topology, The Order Topology, The Product Topology on $X \times Y$, The Subspace Topology, Closed Sets and Limit Points.

Unit II: **[15 Lectures]**

Continuous Functions ,The Product Topology, The Metric Topology, Connected Spaces, Connected Subspaces of the Real Line.

Unit III: **[15 Lectures]**

Components and Local Connectedness,Compact Spaces, Compact Subspaces of the Real Line, Limit Point Compactness.

Unit IV:, **[15 Lectures]**

Local Compactness, TheCountability Axioms,The Separation Axioms, Normal Spaces, The Urysohn Lemma, The UrysohnMetrization Theorem (Only statement and its importance)

Unit V: **[15 Lectures]**

The Tietze Extension Theorem (Only statement and its importance), The Tychonoff Theorem.

Recommended Book:

1. J. R. Munkers, Topology, Second Edition, Pearson Education (Singapore), 2000.

Reference Books:

1. W. J. Pervin, Foundations of General Topology, Academic Press, New York, 1964.
2. J. L. Kelley, General Topology, Springer-Verlag, New York, 1955.
3. S. Willard, General Topology, Addison-Wesley Publishing Company, 1970.
4. K. D. Joshi, Introduction to General Topology,New Age International, 1983.
5. G. F. Simmons, Introduction to Topology and Modern Analysis, McGraw Hill Book Company, New Delhi, 1963.

M. Sc. (Mathematics) (Part I) (Semester II)
(Online Distance Mode)
(Introduced from June 2021 onwards)

Course Code: OMT 204

Title of Paper: Numerical Analysis

Course Outcomes: Upon successful completion of this course, the student will be able to:

1. apply the methods to solve linear and nonlinear equations.
2. find numerical integration and analyze error in computation.
3. solve differential equations using various numerical methods.
4. determine eigen values and eigen vectors of a square matrix.
5. construct LU decomposition of a square matrix.

Unit I

Algebraic and transcendental equations: **[15 Lectures]**

Rate of Convergence of Secant method, Regula Falsi method and Newton-Raphson method.Bairstow method.

System of linear equations: Matrix factorization methods (Doo little reduction, Crout reduction).

Unit II: **[15 Lectures]**

Eigen values and eigenvectors, Gerschgorin theorem, Brauer theorem, Jacobi method for symmetric matrices.

Numerical Integration: Error estimates of trapezoidal and Simpson's numerical integration rule. Gauss-Legendre integration methods (n=1, 2).

Unit III : **[15 Lectures]**

Lobatto integration method (n=2), Radau integration method (n=2) and their error estimates, Runge–Kutta Methods: Second order methods, The coefficient tableau, Third order methods (without proof), order conditions, Fourth order methods (without proof), Implicit Runge–Kutta methods, Stability characteristics.

Unit IV: **[15 Lectures]**

Taylor Series Methods: Introduction to Taylor series methods, Manipulation of power series, An example of a Taylor series solution.

Linear Multistep Methods: Adams methods, General form of linear multistep methods.

Unit V: **[15 Lectures]**

,**Analysis of Linear Multistep Methods**: Predictor– corrector Adams methods, Starting methods Convergence, Consistency, Sufficient conditions for convergence, Stability Characteristics.

Recommended Books :

1. M.K. Jain, S. R. K. Iyengar, R. K. Jain,Numerical Methods for Scientific and Engineering Computation,New Age International Limited Publishers, 6th edition.(For Units 1 and 2)

2. Numerical Methods for Ordinary Differential Equations, J.C. Butcher, John Wiley & Sons Ltd, 2nd edition. (For Units 3 and 4)

Reference Books :-

1. P. Henrici, Discrete Variable Methods in Ordinary Differential Equations, John Wiley &Sons Ltd.
2. S. S. Sastry, Introductory Methods of Numerical Analysis, Prentice Hall of India NewDelhi.

3.M. K. Jain, Numerical Solutions of Differential Equations,Society for Industrial and Applied Mathematics

M. Sc. (Mathematics) (Part I) (Semester II)
(Online Mode)
(Introduced from June 2021 onwards)

Course Code: OMT 205

Title of Course: Partial Differential Equations

Course Outcomes: Upon successful completion of this course, the student will be able to:

1. classify partial differential equations and transform into canonical form
2. solve linear partial differential equations of both first and second order.
3. solve boundary value problems for Laplace's equation, the heat equation, the wave equation by separation of variables, in Cartesian, polar, spherical and cylindrical coordinates.
4. apply method of characteristics to find the integral surface of a quasi linear partial differential equations.
5. establish uniqueness of solutions of partial differential equations.

Unit I: **[15 Lectures]**

Curves and surfaces, First order Partial Differential Equations, classification of firstorder partial differential equations, classifications of Integrals, Linear equations of first order. Pfaffian differential equations, Criteria of Integrability of a Pfaffian differential equation.

Unit II: **[15 Lectures]**

Compatible systems of first order partial differential equations, Charpits method, Jacobi method of solving partial differential equations, Cauchy Problem, Integral surfaces through a given curve for a linear partial differential equations, for a non-linear partial differential equations.

Unit III: [15 Lectures]

Method of characteristics to find the integral surface of a quasi linear partial differential equations.

Second order Partial Differential Equations. Origin of Partial differential equation,wave equations, Heat equation. Classification of second order partial differential equation, Vibration of an infinite string (both ends are not fixed), Physical Meaning of the solution of thewave equation. Vibration of an semi infinite string,

Unit IV: [15 Lectures]

Vibration of a string of finite length, Methodof separation of variables, Uniqueness of solution of wave equation. Heat conduction Problemswith finite rod and infinite rod.

Families to equipotential surfaces, Laplace equation, Solution of Laplace equation,Laplace equation in polar form, Laplace equation in spherical polar coordinates.Kelvin'sinversion theorem.

Unit V: [15 Lectures]

Boundary Value Problems: Dirichlets problems and Neumann problems, Maximum and minimum principles, Stability theorem. Dirichlet Problems and Neumann problems for a circle, for a rectangle and for a upper half plane, Duhamel's Principle.

Recommended Book:

1. T. Amarnath: An Elementary Course in Partial Differential Equations, 2nd edition, Narosa publishing House(2012).

Reference Books:

1. Mark Pinsky: Partial Differential Equations and Boundary-value Problems with Ppplications, AMS,3rd edition(2011).
2. I. N. Sneddon: Elements of Partial Differential Equations, McGraw Hill Int.
3. Fritz John: Partial Differential Equations, Springer(1952).

SHIVAJIUNIVERSITY,KOLHAPUR

Accredited By NAAC with 'A++' Grade

Online Mode

Syllabus For

M.Sc. Mathematics Part-II

Semester III And IV

(Syllabus to be Implemented from June,2022 onwards.)

Syllabi of M.Sc. Mathematics (Part II) (Online Mode)

Title of the course: M.Sc.(Mathematics) (Online Mode)

1) M.Sc. (Mathematics) programme has semester pattern.
2) Structure of the course

The following table gives the scheme of Examination at M.Sc. Mathematics (Part I) according tothe New Syllabus and pattern of Examination.

M.Sc.(Mathematics) Semester - III

Course Code	Title of course	Number of Credits	Duration of Semester end Exam in hrs.	Marks-Term end Exam	Marks-(Internal) Mid Semester Exam	Total marks of Each Subject
OMT 301	Real Analysis	5	3	90	30	120
OMT 302	Field Theory	5	3	90	30	120
OMT 303	Number Theory	5	3	90	30	120
OMT 304	Operational Research I	5	3	90	30	120
OMT 305	Fuzzy Mathematics I	5	3	90	30	120

M.Sc.(Mathematics) Semester - IV

Course Code	Title of course	Number of Credits	Duration of Semester endExam in hrs.	Marks-Term end Exam	Marks-(Internal) Mid Semester Exam	Total marks of Each Subject
OMT 401	Integral Equations	5	3	90	30	120
OMT 402	Advanced Discrete Mathematics	5	3	90	30	120
OMT 403	Algebraic Number Theory	5	3	90	30	120
OMT 404	Operational Research II	5	3	90	30	120
OMT 405	Fuzzy Mathematics II	5	3	90	30	120

M. Sc. (Mathematics) (Part II) (Semester III)
(Online Mode)
(Introduced from June 2022 onwards)

Course Code: OMT 301
Title of Course: Real Analysis
Course Outcomes: Upon successful completion of this course, the student will be able to:

1. Generalise the concept of length of interval.
2. Analyse the properties of Lebesgue measurable sets.
3. Demonstrate the measurable functions and their properties.
4. Understand the concept of Lebesgue integration of measurable functions.
5. Characterize Riemann and Lebesgue integrability.
6. Prove completeness of L^p Spaces.

UNIT I: **[15 Lectures]**
σ - algebra and Borel sets of real numbers, Lebesgue outer measure, The sigma algebra of Lebesgue measurable sets, Outer and inner approximation of Lebesgue measurable sets, Countable additivity.

UNIT II: **[15 Lectures]**
Continuity and Borel- Cantelli lemma,Non measurable Sets, Lebesgue Measurable Functions: Sums, product and composition of measurable functions, Sequential pointwise limits and simple approximation.

UNIT III: **[15 Lectures]**
Littlewood's three principles, Egoroff's theorem, and Lusin's theorem,Lebesgue integration of a bounded measurable function, Lebesgue integration of a non-negative measurable function.

UNIT IV: **[15 Lectures]**
The general Lebesgue integral, Characterization of Riemann and Lebesgue integrabilityDifferentiability of Monotone Functions, Lebesgue's theorem, Functions of bounded variations, Jordan's theorem (Statement only),Absolutely continuous functions,

Unit V: **[15 Lectures]**
integrating derivatives: differentiating indefinite integrals, The L^pSpaces: Normed linear spaces, The inequalities of Young, Hölder and Minkowski, The Riesz- Fischer Theorem.

Recommended Books:
1. H. L. Royden, P.M. Fitzpatrick, Real Analysis, Fourth Edition, PHI Learning Pvt. Ltd., New Delhi, 2010

Reference Books:
1. G. deBarra, Measure Theory and Integration, New Age International (P) Ltd., 1981.
2. I. K. Rana, An Introduction to Measure and Integration, Narosa Book Company, 1997.
3. S. K. Berberian, Measure and Integration, McMillan, New York, 1965.
4. P. K. Jain, V. P. Gupta, Lebesgue Measure and Integration, Wiley Easter Limited, 1986.
5. P. K. Halmos, Measure Theory, Van Nostrand, 1950.

M. Sc. (Mathematics) (Part II) (Semester III)
(Online Mode)
(Introduced from June 2022 onwards)

Course Code: OMT 302

Title of Paper: Field Theory

Course Outcomes: Upon successful completion of this course, the student will be able to:

1) Determine the basis and degree of a field over its subfield.
2) Construct splitting field for the given polynomial over the given field.
3) Find primitive n^{th} roots of unity and n^{th}cyclotomic polynomial.
4) Make use of Fundamental Theorem of Galois Theory and Fundamental Theorem of Algebra to solve problems in Algebra.
5) Apply Galois Theory to constructions with straight edge and compass.

UNIT-I: Algebraic Extensions of fields [15 Lectures]

Adjunction of roots, Algebraic extensions, Algebraically closed fields.

UNIT-II: Normal and Seperable extensions [15 Lectures]

Splitting fields, Normal extensions, Multiple roots, Finite fields.

UNIT-III: Separable extensions, Galois Theory [15 Lectures]

Automorphism groups and fixed fields, Fundamental theorem of Galois theory, Fundamental theorem of algebra.

UNIT-IV: [15 Lectures]

Roots of unity and cyclotomic polynomials, Cyclic extensions

Unit V: [15 Lectures]

Applications of Galois theory

Polynomials solvable by radicals, Symmetric functions, Constructions by ruler and compass.

Recommended Book(s):

1. Bhattacharya, Jain and Nagpal, Basic Abstract Algebra, 2nd edition, Cambridge University Press, UK.(Asian edition) 2005.

Reference Books:

1. Nathan Jacobson, Basic Algebra I, second edition, W. H. Freeman and company, New York
2. I. N. Herstein, Topics in Algebra, Wiley Eastern Ltd.
3. U. M. Swamy, A. V. S. N. Murthy, Algebra: Abstract and Modern, Pearson Education, 2012
4. John Fraleigh, A first course in Abstract Algebra (3rd edition) Narosa publishing house, New Delhi
5. I. T. Adamson, Introduction to Field Theory, second edition, Cambridge University Press,1982.
6. M. Artin, Algebra, PHI, 1996.
7. Ian Stewart, Galois Theory, CRC Publication.

M. Sc. (Mathematics) (Part II) (Semester III)
(Online Mode)
(Introduced from June 2022 onwards)

Course Code: OMT 303

Title of Paper: Number Theory

Course Outcomes: Upon successful completion of this course, the student should be able to:

1. Learn more advanced properties of primes and pseudo primes.
2. Apply Mobius Inversion formula to number theoretic functions.
3. Explore basic idea of cryptography.
4. Understand concept of primitive roots and index of an integer relative to a given primitive root.
5. Derive Quadratic reciprocity law and its apply to solve quadratic congruences.

Unit I: **[15 Lectures]**

Review of divisibility : The division algorithm, G.C.D., Euclidean algorithm, Diophantine equation ax + by = c. Primes and their distribution : Fundamental theorem of Arithmetic.

Unit II: **[15 Lectures]**

The Goldbach Conjecture,Congruences : Properties of Congruences, Linear congruences, Special divisibility tests. Fermat's theorem : Fermat's factorization method, Little theorem, Wilsons theorem.

Unit III: **[15 Lectures]**

Number theoretic functions : The functions τ and σ. The Mobius Inversion formula, The greatest integer function.Euler's Generalization of Fermat's theorem: Euler's phi function, Euler's theorem, properties of phi function.

Unit IV: **[15 Lectures]**

An application to Cryptography. Primitive roots : The order of an integer modulo n. Primitive roots for primes, composite numbers having primitive roots, The theory of Indices.

Unit V: **[15 Lectures]**

The Quadratic reciprocity law : Eulerian criteria, the Legendre symbol and its properties, quadratic reciprocity, quadratic reciprocity with composite moduli .

Recommended Book:

1. D. M. Burton : Elementary Number Theory, Seventh Ed. MacGraw Hill Education(India)Edition 2012, Chennai.

Reference Books:

1. S. B. Malik :Basic Number theory, Vikas publishing House.
2. George E. Andrews : Number Theory, Hindustan Pub. Corp.(1972).
3. Niven, Zuckerman : An Introduction to Theory of Numbers. John Wiley & Sons.
4. S. G. Telang , Number Theory, Tata Mc.Graw-Hill Publishing Co., New Delhi.
5. M. B. Nathanson, Methods in Number Theory, Springer(2009).

M. Sc. (Mathematics) (Part II) (Semester III)
(Online Mode)
(Introduced from June 2022 onwards)

Course Code: OMT 304

Title of Course: Operations Research I

Course Outcomes-: Upon successful completion of this course, the student will be able to:-

1. Identify Convex set and Convex functions.
2. Construct linear integer programming models and discuss the solution techniques,
3. Formulate the nonlinear programming models,
4. Propose the best strategy using decision making methods,
5. Solve multi –level decision problems using dynamic programming method.

Unit I : [15 Lectures]

Convex set and their properties: Lines and hyper planes, convex set, Important Theorems, Polyhedral convex sets, Convex combination of vectors, Convex hull, Convex polyhedron, Convex cone, Simple and convex functions. General formulation of linear programming, Matrix form of linear programming problem, Definitions of standard linear programming problem, Fundamental Theorem of linear programming, Simplex method,.

Unit II: [15 Lectures]

Computational procedure of simplex method, Problem of degeneracy and method to resolve degeneracy,Revised simplex method in standard form I, Duality in linear programming, duality theorems, Dual simplex method

Unit III: [15 Lectures]

Integer linear programming, Gomory's cutting plane method, Branch and bound method.Dynamic programming: Bellman's principle of optimality, Solution of problem with a finite number of stages.

Unit IV : [15 Lectures]

Application of dynamic programming in production, Inventory control and linear programming,Non – linear programming unconstrained problems of maximum and minimum.

Unit V: [15 Lectures]

Lagrangian method ,Quadratic programming, Kuhn Tucker necessary and sufficient condition, Wolfe method, Beale's method.

Recommended Books :

1. S.D. Sharma : Operations Research , Kedar Nath Ram Nath and Co.
2. J K Sharma: Operations Research Theory and Applications, Mac Millan Co.

Reference Books :

1. Kanti Swarup ,P.K.Gupta and Manmohan : Operations Research , S. Chand & Co.
2. Hamady Taha : Operations Research : Mac Millan Co.
3. S.D. Sharma: Linear Programming ,Kedar Nath Ram Nath and Co.
4. S.D. Sharma : Nonlinear and Dynamic Programming , Kedar Nath Ram Nath and Co. Meerut.
5. R. K. Gupta : Operations Research, Krishna Prakashan Mandir , Meerut.
6. G. Hadley : Linear Programming , Oxford and IBH Publishing Co.

M. Sc. (Mathematics) (Part II) (Semester III)
(Online Mode)
(Introduced from June 2022 onwards)

Course Code: OMT 305

Title of Paper: Fuzzy Mathematics-I

Course Outcomes: Upon successful completion of this course, the student will be able to:

1. Acquire the knowledge of notion of crisp sets and fuzzy sets,
2. Understand the basic concepts of crisp set and fuzzy set,
3. Develop the skill of operation on fuzzy sets and fuzzy arithmetic,
4. Demonstrate the techniques of fuzzy sets and fuzzy numbers.
5. Apply the notion of fuzzy set, fuzzy number in various problems.

Unit I: **[15 Lectures]**

Fuzzy sets and crisp sets, examples of fuzzy sets, types of fuzzy sets, standard operations, cardinality, degree of subset hood, level cuts and its properties, **r**epresentation of fuzzy sets, decomposition theorems, extension.

Unit II: **[15 Lectures]**

Principle properties of direct and inverse images of fuzzy sets,Operations on fuzzy sets, types of operations, fuzzy complement, equilibrium and dual point.

Unit III: **[15 Lectures]**

Increasing and decreasing generators, fuzzy intersection: t-norms.Fuzzy union t-conorms, characterization theorem of t-conorm, combination of operators.

Unit IV: **[15 Lectures]**

Aggregation operations, ordered weighted averaging operations,Fuzzy numbers, characterization theorem,

Unit V: **[15 Lectures]**

linguistic variables, arithmetic operations on intervals, arithmetic operations on fuzzy numbers, lattice of fuzzy numbers, fuzzy equations.

Recommended Books:

1. George J. Klir, Bo Yuan, Fuzzy sets and Fuzzy Logic. Theory and Applications, PHI, Ltd.2000

Reference Books:-

1. M.Grabish, Sugeno, and Murofushi Fuzzy Measures and Integrals: Theory and Applications,PHI, 1999.
2. H.J.Zimmerermann, Fuzzy Set Theory and its Applications, Kluwer, 1984.
3. M. Hanss, Applied Fuzzy Arithmetic, An Introduction with EngineeringApplications, Springer-Verlag Berlin Heidelberg 2005.
4. M. Ganesh, Introduction to Fuzzy Sets & Fuzzy Logic; PHI Learning Private Limited, New Delhi 2011.

Bojadev and M. Bojadev, Fuzzy Logic and Application, World Scientific Publication Pvt.Ltd. 2007.

M. Sc. (Mathematics) (Part II) (Semester IV)
(Online Distance Mode)
(Introduced from June 2022 onwards)

Course Code: OMT 401

Title of Paper: Integral Equations

Course Outcomes: Upon successful completion of this course, the student will be able to:

1. Classify the linear integral equations and demonstrate the techniques of converting the initial and boundary value problem to integral equations and vice versa.
2. Develop the technique to solve the Fredholm integral equations with separable kernel.
3. Develop and demonstrate the technique of solving integral equations by successive approximations, using Laplace and Fourier transforms
4. To analyze the properties of symmetric kernel.
5. To prove Hilbert Schmidt Theorem and solve the integral equation by applying it.

UNIT– I **[15 Lectures]**

Classification of linear integral equations, conversion of initial value problem to Volterra integral equation, conversion of boundary value problem to Fredholm integral equation, separable kernel, Fredholm integral equation with separable kernel, Fredholm alternative.

UNIT –II **[15 Lectures]**

Homogeneous Fredholm equations and eigen functions.Solutions of Fredholm integral equations by: Successive approximations method, successive substitution method, Adomian decomposition method, modified decomposition method, resolvent kernel of Fredholm equations and its properties.

UNIT –III **[15 Lectures]**

Solutions of Volterra integral equations, successive approximations method, Neumann series, successive substitution method,Solution of Volterra integral equations by Adomian decomposition method and the modified decomposition method, resolvent kernel of Volterra equations and its properties, convolution type kernels.

UNIT – IV **[15 Lectures]**

Applications of Laplace and Fourier transforms to solutions of Volterra integral equations, symmetric kernels**,** fundamental properties of eigen values and eigen functions for symmetric kernels, expansion in eigen functions and bilinear form.Hilbert Schmidt Theorem and its consequences, solution of symmetric integral equations.

Unit V: **[15 Lectures]**

Operator method in the theory of integral equations, solution of Volterra and Fredholm integro-differential equations by Adomian decomposition method. Green's function: Definition, construction of Green's function and its use in solving boundary value problems.

Recommended Books:

1. R. P. Kanwal, Linear Integral Equation: Theory and Technique, Birkhauser 2012.
2. Abdul-Majid Wazwaz, Linear and Nonlinear Integral Equations-Methods and Applications, Springer, 2011

Reference Books:

1. L. G. Chambers, Integral Equations- A Short Course, International Text Book Company, 1976.
2. M. A, Krasnov, et.al. Problems and exercises in Integral equations, Mir Publishers, 1971.
3. J. A. Cochran, The Analysis of Linear Integral Equations, Mc Graw Hill Publications, 1972.
4. C. D. Green, Integral Equation Methods, Thomas Nelson and sons, 1969.

M. Sc. (Mathematics) (Part II) (Semester IV)
(Online Mode)
(Introduced from June 2022 onwards)

Course Code: OMT 402

Title of Course: Advanced Discrete Mathematics

Course Outcomes: Upon successful completion of this course, the student will be able to:

1. Classify the graphs and apply to real world problems.
2. Simplify the graphs using matrix.
3. Study Binomial theorem and use to solve various combinatorial problems.
4. Simplify the Boolean identities and apply to switching circuits.
5. Locate and use information on discrete mathematics and its applications.

Unit I **[15 Lectures]**

Graph: Definition, examples, isomorphism, simple graph, bipartite graph, complete bipartite graph, vertex degrees, regular graph, sub-graphs, complement of a graph, self complementary graph, paths and cycles in a graph, the matrix representation of a graph, fusion.

Unit II **[15 Lectures]**

Definition and simple properties of a treeBridges, spanning trees, cut vertices and connectivity, Euler Tours and Hamiltonian cycles, Fleury's Algorithm, Hamiltonian graphs.

Unit III **[15 Lectures]**

Plane and planar graphs, Euler's formula,Principle of inclusion and exclusion, Pigeonhole principle, permutations and combinations, Binomial theorem, discrete numeric functions, manipulation of numeric functions, generating functions, linear recurrence relations with constant coefficients.

Unit IV **[15 Lectures]**

particular solutions of linear recurrence relations, total solutions, solution by the method of generating function,Posets: Definition, examples, Hasse diagrams of posets, supremum and infimum, isomorphic ordered sets, duality. Lattices: Definition, examples, sublattices. Ideals: Definition, examples.

Unit V: **[15 Lectures]**

Bounded lattices, distributive lattices, modular lattices, complemented lattices, Boolean algebra, basic definitions, basic theorems, Boolean algebras as lattices, CNF, DNF, applications of Boolean algebra to switching circuit.

Recommended Books:

1. John Clark and Derek Holton , A first look at Graph Theory, Allied Publishers Ltd.,1991.
2. C. L. Liu, D. P. Mohapatra, Elements of Discrete Mathematics, Tata McGraw Hill Pvt Ltd, 1985.
3. G. Gratzer, General Lattice Theory, Birkhauser,2002.

Reference Books:

1. SeymonLipschutz and Mark Lipson, Discrete Mathematics (second edition) Tata Mc Graw Hill Publishing Company Ltd. New Delhi.
2. Garrett Birkhoff : Lattice Theory, American mathematical society,1940.
3. Richard A. Brualdi: Introductory Combinatorics,Pearson,2004.

M. Sc. (Mathematics) (Part II) (Semester IV)
(Online Mode)
(Introduced from June 2021 onwards)

Course Code: OMT 403

Title of Paper: Algebraic Number Theory.

Course Outcomes: Upon successful completion of this course, the student will be able to

1. Deal with algebraic numbers , algebraic integers and its applications,
2. Concept of lattices and geometric representation of algebraic numbers.
3. Understand the concept of fractional ideals.
4. Relate Finitely generated abelian groups and modules
5. Derive Minkowski's theorem.
6. Compute class groups and class numbers.

Unit I: **[15 Lectures]**

Revision of basic module theory, Fundamental concepts and results, Free modules and matrices, Direct sums of modules, Finitely generated modules over a P.I.D., Equivalence of matrices with entries in a P.I.D..

Unit II: **[15 Lectures]**

Structure theorem for finitely generated modules over a P.I.D. and applications to abelian groups,Algebraic Numbers, Quadratic and cyclotomic fields, Factorization into irreducibles.

Unit III: **[15 Lectures]**

Euclidean quadratic fields,Prime factorization of ideals.

Unit IV: **[15 Lectures]**

Prime factorization of Lattices, Minkowski's theorem,Geometric Representation of algebraic numbers.

Unit V: **[15 Lectures]**

Class groups and class numbers, computational methods.

Recommended Books:

1. N.Jacobson, Basic Algebra - I, Hindustan Publishing Corporation (India), Delhi (Unit-I)
2. I.N. Stewart and D.O. Tall, Algebraic Number Theory and Fermat's Last Theorem,2015,CRC press. (Chapters 2 to 10) (Unit-II to Unit-IV)

Reference Books:

1. Algebraic Number Theory : Mathematical Pamphlet, TIFR, Bombay .
2. Paulo Ribenboim, Classical Theory of Algebraic Numbers, Springer , New York(2001).
3. N. S. Gopalkrishnan, University Algebra, New Age International(P) Ltd. Publishers.
4. Ian Stewart, Galoi Theory, CRC press(2015).

Harry Pollard, The Theory of Algebraic Numbers, The Mathematical Association of America.

M. Sc. (Mathematics) (Part II) (Semester IV)
(Online Mode)
(Introduced from June 2022 onwards)

Course Code: MT 404

Title of Paper: Operations Research – II

Course Outcome-: Upon successful completion of this course, the student will be able to:-

1.Decide policy for replacement.

2.Calculate economic lot size.

3.DerivePoission distribution theorem and compute attributes of distribution model. 4.construct Shannon Fano codes.

4.Identify optimal path by using CPM and PERT

Unit I **[15 Lectures]**

Replacement problems: failure mechanism of items, replacement policy for items whose maintenance cost increases with time and money value is constant, Money value, Present worth Factor, Discount rate, replacement policy for items whose maintenance cost increases with time and money value changes with constant rate.

Unit II **[15 Lectures]**

Group replacement of items that fail completely.Inventory : cost involved in inventory problems, variables in inventory problem, symbols in inventory, concept of EOQ , Model I (a) The economic lot size system with uniform demand, Model I (b) The economic lot size with different rates of demand in different cycles, Model I (c) The economic lot size with finite rate of replenishment ,(EOQ production model) EOQ model with shortages.

Unit III **[15 Lectures]**

Model II (a) The EOQ with constant rate of demand , scheduling , time constant, Model II (c) The production lot size model with shortages , probabilistic inventory models, instantaneous demand , no set up cost model, Model VI (a) Discrete case , Model VI (b) continuous case, Queuing theory, queuing systems, queuing problems, transient and steady states. traffic intensity.

Unit IV **[15 Lectures]**

Probability distributions in queuing system, Poisson process, properties , exponential process , classification of queuing models , Model I : (M/M/I) : (infinity / FCFS)

Model II (a) : General Erlang Queuing model Information theory : Communication process, quantitative measure of information , a binary unit of information, measure of uncertainty: entropy , basic properties of entropy function (H), joint and conditional entropies

Unit V: **[15 Lectures]**

Uniqueness theorem, channel capacity ,efficiency and redundancy , encoding , Shannon Fano encoding procedure ,PERT / CPM: Applications of PERT / CPM techniques , network diagram, representations, rules for constructing the network diagram, determination of the critical path.

Recommended Books :

1.S.D. Sharma : Operations Research , KedarNath Ram Nath and Co.

2.J K Sharma: Operations Research :Theory and Applications, Mac Millan Co.

Reference Books :

1. KantiSwarup ,P.K.Gupta and Manmohan : Operations Research , S. Chand & Co.
2. Hamady Taha : Operations Research : Mac Millan Co.
3. S.D. Sharma: Linear Programming ,KedarNath Ram Nath and Co.
4. S.D. Sharma : Nonlinear and Dynamic programming KedarNath Ram Nath and Co. Meerut.

5. R.K.Gupta : Operations Research, Krishna PrakashanMandir , Meerut.
6. G.Hadley : Linear Programming , Oxford and IBH Publishing Co.

M. Sc. (Mathematics) (Part II) (Semester IV)
(Online Mode)
(Introduced from June 2022 onwards)

Course Code: OMT 405

Title of Paper: Fuzzy Mathematics-II

Course Outcomes: Upon successful completion of this course, the student will be able to:

1. Acquire the concept of fuzzy relations.
2. Understand the basic concepts of fuzzy logic and fuzzy algebra.
3. Develop the skills of solving fuzzy relation equations.
4. Construct approximate solutions of fuzzy relation equations. 5.solve problems in Engineering and medicine.

Unit I **[15 Lectures]**

Projections and cylindrical extensions, binary fuzzy relations on single set, fuzzy equivalence relations, fuzzy compatibility relations, fuzzy ordering relations.

Unit II **[15 Lectures]**

fuzzy morphisms sup-i composition and inf-wi composition.Fuzzy relation equations, problem partitioning, solution methods.

Unit III **[15 Lectures]**

fuzzy relational equations based on sup-i and inf-wi compositions, approximate solutions,Fuzzy propositions, fuzzy quantifiers, linguistic hedges.

Unit IV **[15 Lectures]**

inference from conditional fuzzy propositions, qualified and quantified propositions,Fuzzy algebra,

Unit V: **[15 Lectures]**

Fuzzy groups and fuzzy rings and their basic properties

Recommended Books:

1. George J Klir, Bo Yuan, Fuzzy Sets and Fuzzy Logic. Theory and applications,PHI.Ltd. (2000)
2. John Mordeson, Fuzzy Mathematics,Springer,2001

Reference Books:

1.M.Grabish, Sugeno, and Murofushi, Fuzzy Measures and Integrals: Theory and Applications PHI, 1999.
2. H.J.Zimmerermann, Fuzzy set :Theory and its Applications, Kluwer, 1984.
3.M. Ganesh, Introduction to Fuzzy sets & Fuzzy Logic; PHI Learning Private Limited, New Delhi. 2011.