DECISION MAKING THROUGH FUZZY RELATION

R. R. Mudholkar[1], P. A. Kadam[1], M. D. Hanamane[2],

R. A. Nanaware[1], S. S. Nirmale[1], P. P. Murkute[1]

Department of Electronics, Shivaji University, Kolhapur[1]

Department of Electronics, K.W.C. College, Sangli[2]

ABSTRACT:

In day-to-day life one way or other decisions are to be made and actions are to be taken. With modernization of life style the world is becoming more and more uncertain and Decision Making is becoming a subtle task. Further it complicates if more than one alternative option emerge. This needs the optimization of decisions to arrive at best one. There exit classical theories to develop good decisions but they have limitations in handing uncertainties adequately. Fuzzy Set Theory aims to model and handle uncertainties arising due to imprecise data, incomplete information, ambiguity and vagueness in the natural language. The paper presents an alternative approach of deriving decisions founded on the concepts of Fuzzy Relation. This approach has ability to intermingle the Decision Making with Decision Optimization in one attempt. An attempt is made to illustrate the potentiality of Fuzzy Set Theory in the process of Decision Making.

Keywords: Decision-Making, Fuzzy Relation, Uncertainty.

I. INTRODUCTION:

Decision-Making under uncertainty is as old as mankind. With increasing sophistication and modernization of life style Decision Making is becoming more scientific and machine dependent. People are looking for a support to ease the Decision Making process. Since number of alternative decisions may exits, going for good choice imposes another task of Decision Making called optimizing the decisions.

The subject of Decision Making is the scientific study of how, which and when decisions are actually be made and how can be made better and more successful decisions. It is of great concerned with almost every field. However this has been mainly focussed in the area of management in which good policies have to be emerged from decisions made overcoming contradictions and uncertainties. Decision Making can put in simple words as any choice or selection of available alternatives made toward best outcomes. This has significant role in the fields of soft social sciences to hard disciplines of natural sciences and engineering.

So far the theories developed for making good decisions followed by optimization based on the classical approaches like Probability Theory and Game Theories are undergoing paradigmatic changes especially in the handling and modeling the uncertainties arising due to imprecision, non-specificity, vagueness and inconsistency, which are considered unscientific. However with the emergence of Fuzzy Set Theory followed by Fuzzy Logic the Researchers realized the great utility of uncertainties in Science and Engineering fields including support theories for Decision Making. Application of Fuzzy Set Theory in the area of Decisions Support Systems is broadly considered as a Fuzzification of classical theories of Decision Making. The Fuzzy Approach in this regard thus attempts to deal with the vagueness, ambiguity, non-specificity inherent in human formulation of preferences, constraints and goals. Using two-valued logic for optimizing the decisions and goals has surmountable difficulties. But Fuzzy based Decision Support greatly overcomes the difficulties encountered in the classical Decision Support through two approaches –

1. Based on Fuzzy Sets
2. Based on Fuzzy Relation

In either approaches goals are achieved even if the outcomes of decisions can be characterized only approximately. In other words the decisions are made under uncertainty. This is the foundation of fuzzy based Decision Support systems.

In the following text we present application potential of Fuzzy Relation in deriving decisions under uncertainty and illustrate this with general example.

II. METHODOLOGY:

The decision-making using Fuzzy Relation involves various steps. First step is to study and understand the functional relationship amongst various problem variables and partition them as input and output variables. The next step is to define the fuzzy sets for input and output variables. Next step is to develop a fuzzy relation between them. Take the input condition and apply compositional rules of fuzzy inference on it. Fuzzy inference gives a decision outcome. Finally the outcome decision is interpreted for action to be taken. Fig. 1 shows the steps involved in Decision Making by Fuzzy Relation.

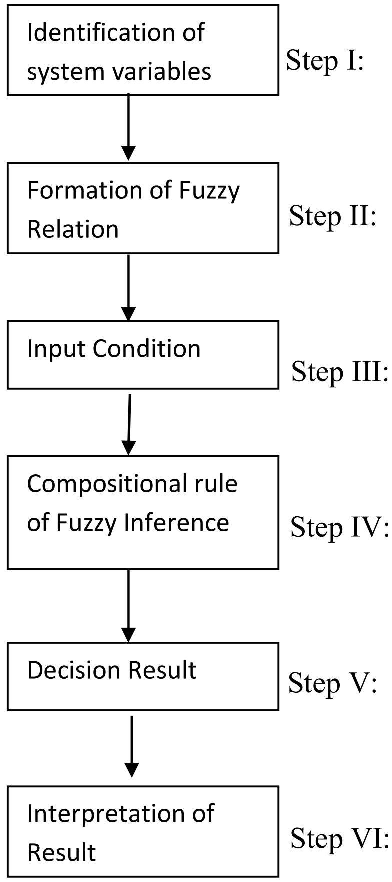

Fig 1: Decision Making by Fuzzy

Fuzzy Relation:

In real life the things are not related absolutely to each other. In fact majority of relations are approximate in nature. The relations between the things can then only be defined on the varying degrees of relationship. These degrees indicate the strength of the relation between them. A fuzzy relation 'R' between sets 'A' and 'B' can be put as -

$$R = \int_{AxB} \mu_R(a,b)/(a,b)$$

where μ_R represents degree of relationship. The fuzzy relation in the matrix form is given as-

$$A = \{a_1 , a_2 , a_{3,} \ldots , a_n\}$$

$B = \{b_1, b_2, b_3, \ldots, b_m\}$

$$R = \begin{array}{c} \\ a_1 \\ a_2 \\ \ldots \\ a_n \end{array} \begin{array}{c} \begin{array}{cccc} b_1 & b_2 & \ldots & b_m \end{array} \\ \begin{bmatrix} \mu_R(a_1,b_1) & \mu_R(a_1,b_2) & \ldots & \mu_R(a_1,b_m) \\ \mu_R(a_2,b_1) & \mu_R(a_2,b_2) & \ldots & \mu_R(a_2,b_m) \\ \ldots & \ldots & \ldots & \ldots \\ \mu_R(a_n,b_1) & \mu_R(a_n,b_2) & \ldots & \mu_R(a_n,b_m) \end{bmatrix} \end{array}$$

How 'close' the data set A = {1, 2, 3, 4} to data set B = {2,4,6,8} is, can be represented in the Fuzzy Relation as-

$$R = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} 2 & 4 & 6 & 8 \end{array} \\ \begin{bmatrix} 0.8 & 0.5 & 0.2 & 0 \\ 1 & 0.6 & 0.3 & 0 \\ 0.4 & 0.7 & 0.2 & 0.1 \\ 0.2 & 1 & 0.5 & 0.2 \end{bmatrix} \end{array}$$

To obtain a fuzzy set from a Fuzzy Relation and vice versa respectively projection operation and cylindrical extension are performed.

$$proj[R;A] = \int_A (\max_b \mu_R(a,b)) / a \qquad proj[R;B] = \int_B (\max_a \mu_R(a,b)) / b$$

$$c(A) = \int_{AxB} \mu_A(a)/(a,b) \qquad c(B) = \int_{AxB} \mu_B(b)/(a,b)$$

Outcome of Fuzzy Decision is computed using compositional rule of inference as –

$$\mu_{AoR}(b) = \max_{a \in A} [\mu_A(a) \wedge \mu_R(a,b)]$$

III. CASE STUDY:

Mr. Sameer is an industrialist and is willing to have a partner in his business. He is qualifying the person as partner if "they both can pay off the payments for the New Plant they will buy, within 10 years." Mr. Rohit, Mr. John, Mrs. Pooja are willing to work with Mr. Samer. But to choose one of these aspirants Mr. Sameer need to know their financial conditions. As the financial condition can't be known from the information provided by the individuals Mr. Sammer needs to predict it from the cars the individuals own. Their cars prices are –

Table 1. Person name and price of their own car

Name	**Car's Price**
Mr. Rohit	4,00,000
Mr. John	2,50,000
Mrs. Pooja	1,00,000

From the table.1 data, Mr. Rohit looks to be an obvious choice as partner. Let's find out the perfect match by using Fuzzy Relation and composition.

The rules defined are

RULE 1:

IF THE CAR IS *EXPENSIVE,*

THEN INCOME IS *HIGH.*

RULE 2:

IF INCOME IS *HIGH,*

THEN MORTGAGE WILL BE PAID IN *SHORT* PERIOD.

Let's define the Fuzzy Sets –

Car's price = {cheap, middle, expensive}

Income = {low, average, high}

Mortgage payment period = {short, average, long}

Let's define a relation from RULE 1 between car price and income .

$$R = (\text{Car}) \begin{matrix} \\ cheap \\ middle \\ \exp ensive \end{matrix} \overset{(\text{Income})}{\begin{matrix} low & middle & high \\ \end{matrix}} \begin{bmatrix} 0.7 & 0.3 & 0.4 \\ 0.3 & 0.9 & 0.5 \\ 0.2 & 0.3 & 0.7 \end{bmatrix}$$

The other Fuzzy Relation from RULE 2 between income and Mortgage payment can be given as-

$$S = (\text{Income}) \begin{array}{c} \\ \text{low} \\ \text{average} \\ \text{high} \end{array} \overset{\text{(Mortgage Payment)}}{\begin{array}{ccc} \text{short} & \text{average} & \text{long} \\ \end{array}} \begin{bmatrix} 0 & 0.2 & 1.0 \\ 0.3 & 0.6 & 0.6 \\ 0.9 & 0.3 & 0.4 \end{bmatrix}$$

To know the income status of all of them from their car's prices we will carry out the composition of the fuzzy sets R and S i.e (RoS).

$$RoS = \begin{array}{c} \\ \textit{cheap} \\ \textit{middle} \\ \textit{expensive} \end{array} \begin{array}{c} \textit{low} \quad \textit{middle} \quad \textit{high} \\ \begin{bmatrix} 0.7 & 0.3 & 0.4 \\ 0.3 & 0.9 & 0.5 \\ 0.2 & 0.3 & 0.7 \end{bmatrix} \end{array} \;o\; \begin{array}{c} \\ \text{low} \\ \text{average} \\ \text{high} \end{array} \begin{array}{c} \text{short} \quad \text{average} \quad \text{long} \\ \begin{bmatrix} 0 & 0.2 & 1.0 \\ 0.3 & 0.6 & 0.6 \\ 0.9 & 0.3 & 0.4 \end{bmatrix} \end{array}$$

$$RoS = \begin{array}{c} \\ \text{cheap} \\ \text{middle} \\ \text{expensive} \end{array} \begin{array}{c} \text{short} \quad \text{average} \quad \text{long} \\ \begin{bmatrix} 0.4_{\text{'a'}} & 0.3 & 0.7 \\ 0.5 & 0.6 & 0.6 \\ 0.7 & 0.3 & 0.4 \end{bmatrix} \end{array}$$

Note 1:'a' = max [min(0.7,0),min(0.3,0.3),min(0.4,0.9)]

= max [0,0.3,0.4]

= 0.4

Now let's categorize individual's car's price in terms of Fuzzy Matrix as-

$$A = \begin{array}{c} \\ \text{Mr. Rohit} \\ \text{Mr. John} \\ \text{Mrs. Pooja} \end{array} \begin{array}{c} \text{cheap} \quad \text{middle} \quad \text{expensive} \\ \begin{bmatrix} 0.1 & 0.3 & 0.9 \\ 0.2 & 0.9 & 0.4 \\ 0.9 & 0.3 & 0.1 \end{bmatrix} \end{array}$$

The fact that car of Mr. Rohit is expensive or that of Mrs. Pooja is cheap or "least" expensive can be interpreted from the above matrix. The mortgage paying periods for each individual can be inferred by combining the matrix A and matrix (RoS) under the compositional rule of inference as-

Mr. Rohits mortgage period can infer to be:

$$\begin{array}{c} \text{cheap} \quad \text{middle} \quad \text{expensive} \\ \begin{bmatrix} 0.9 & 0.3 & 0.1 \end{bmatrix} \end{array} \;o\; \begin{array}{c} \\ \text{cheap} \\ \text{middle} \\ \text{expensive} \end{array} \begin{array}{c} \text{short} \quad \text{average} \quad \text{long} \\ \begin{bmatrix} 0.4 & 0.3 & 0.7 \\ 0.5 & 0.6 & 0.6 \\ 0.7 & 0.3 & 0.4 \end{bmatrix} \end{array}$$

The result can be obtained as described in the above composition operation as-

(Mortgage)

Mr. Rohit:-

short	average	long
0.7	0.3	0.4

Mr. John's mortgage period can inferred to be:

$$\begin{bmatrix} 0.2 & 0.9 & 0.4 \end{bmatrix}_{\text{cheap, middle, expensive}} \circ \begin{array}{l} \text{cheap} \\ \text{middle} \\ \text{expensive} \end{array} \overset{\begin{array}{ccc}\text{short} & \text{average} & \text{long}\end{array}}{\begin{bmatrix} 0.4 & 0.3 & 0.7 \\ 0.5 & 0.6 & 0.6 \\ 0.7 & 0.3 & 0.4 \end{bmatrix}}$$

The result can be obtained as described in the above composition operation as-

(Mortgage)

Mr. John :-

short	average	long
0.5	0.6	0.4

Mrs. Poojas mortgage period can inferred to be:

$$\begin{bmatrix} 0.9 & 0.3 & 0.1 \end{bmatrix}_{\text{cheap, middle, expensive}} \circ \begin{array}{l} \text{cheap} \\ \text{middle} \\ \text{expensive} \end{array} \overset{\begin{array}{ccc}\text{short} & \text{average} & \text{long}\end{array}}{\begin{bmatrix} 0.4 & 0.3 & 0.7 \\ 0.5 & 0.6 & 0.6 \\ 0.7 & 0.3 & 0.4 \end{bmatrix}}$$

The result can be obtained as described in the above composition operation as-

Mrs. Pooja :-

short	average	long
0.4	0.3	0.7

IV. RESULT AND DISCUSSION:

	short	average	long
Mr.Rohit	0.7	0.3	0.4
Mr. John	0.5	0.6	0.4
Mrs. Pooja	0.4	0.3	0.7

The final decision matrix suggests that the good choice for Mr. Sameer as a business partner is Mr. Rohit as his grade under short duration is highest i.e. '0.7' meaning Mr. Rohit will take shorter time to invest money in the business. Next choice could be Mr. John whereas Mrs. Pooja looks to be the worst choice in the conditions considered herein. This Fuzzy Relational Approach can be extended to similar problems of decision-making! Decision-Making is an integral part of day-to-day life. Therefore systematic approach towards better decisions followed by optimization of outcomes is the prime need of

Individuals, Society, Organizations, Industry, and Policy Makers. In present paper this point is highlighted with the potentiality of Fuzzy Relation in deriving decisions under all sorts of uncertainties. The Case Study shows the easiness of approach, which is founded on non-mathematical background rendered by Fuzzy Set Theory. This has emerged out as a new trend in the world of computer based Decision-Making.

V. REFERENCES:

1. Lootsma F. A., "Fuzzy Logic for Planning and Decision making", Kluwer Academic publication. (1997).

2. HACI AKTAS, "ON FUZZY RELATION AND FUZZY QUOTIENT GROUPS", International Journal of Computational Cognition Volume 2, Number 2, Pages 71–79, June 2004.

3. Martin ˇStˇepniˇcka and Lenka Noskov´, "Fuzzy Relation Equations – New Solutions and Solvability Criteria" Institute for Research and Applications of Fuzzy Modeling, University of Ostrava.

4. B.ˇSeˇselja and Andreja Tepavˇcevi´c, "Cut equivalence of fuzzy relations", Department of Mathematics and Informatics, University of Novi Sad.

5. B. ˇSeselja, A. Tepavˇcevi´c, "A note on natural equivalence relation on fuzzy power set" University of Novi Sad, preprint

6.Yeh R.T, S.Y.Bang, L.A. Zadeh, S. Fu, M.Shimula (Eds.), Fuzzy relations, Fuzzy graphs and their applications to cluster analysis, in;, Fuzzy sets and Their Applications, Academic press New York,1975.

7. Klir G. J. and BoYuan "Fuzzy Set and Fuzzy Logic –Theory and Application", , PHI of India (2000).

8. Kauzo Tanaka "An Introduction to Fuzzy Logic for Practical Application", Springer (1996).

9. Zadeh L.A., "Fuzzy sets, Inform and Control" 8(1965) 338-365

10. Klawon, F, Nov´ak, V., Perfilieva: Fuzzy points, fuzzy relations and fuzzy functions. In I(eds.): Discovering the World with Fuzzy Logic. Studies in Fuzzines and Soft Computing. Springer Physica-Verlag, Heidelberg New York (2000) 431–453.