Department of Mathematics, Shivaji University,
LINEAR ALGEBRA

1. (a) (Two marks each points) Chose the correct answer and rewrite the sentence.

i. $L(S)$ is subspace of V. If S, T are subsets of V then $L(S \bigcup T) = \cdots\cdots\cdots$.

(A) $L(S \bigcap T)$ (B) $L(S) \bigcup L(T)$ (C) $L(S) + L(T)$ (D) $L(S + T)$

ii. If V is finite dimensional over F then V is isomorphic to F^n for unique integer $\cdots\cdots\cdots$.

(A) m (B) n (C) $|F|$ (D) $|V|$

iii. Any two finite dimensional vector spaces over F of the same dimension are $\cdots\cdots\cdots$.

(A) homomorphic (B) automorphic (C) epimorphic (D) isomorphic

iv. Hom(V, W) forms $\cdots\cdots\cdots$ over F.

(A) Ring (B) field (C) vector space (D) Module

v. If V is of dimensions m, over F, then Hom(V, F) is of dimension $\cdots\cdots\cdots$ over F .

(A) 1 (B) $|F|$ (C) $|V|$ (D) m

vi. If V is finite dimensional over F then there is $\cdots\cdots\cdots$ of V onto $\hat{\hat{V}}$.

(A) homomorphism (B) automorphism (C) epimorphism (D) isomorphism

vii. W is subspace of V a finite dimensional vector space over F then $A(W)$ subspace of $\cdots\cdots\cdots$.

(A) W (B) $\hat{V}$ (C) $\hat{\hat{V}}$ (D) V

viii. Let W_1, W_2 be subspaces of V a finite dimensional vector space over F. $A(W_1 + W_2) = \cdots\cdots\cdots$.

(A) $A(W_1 \bigcup W_2)$ (B) $A(W_1 \bigcap W_2)$ (C) $A(W_1) \bigcap A(W_2)$ (D) $A(W_1) \bigcup A(W_2)$

ix. In an inner product space V, we have $(u, u) \geq 0$ and $(u, u) = 0$ if and only if $\cdots\cdots\cdots$.

(A) $u = 0$ (B) $v = 0$ (C) $V = \{0\}$ (D) $u^2 = 0$

x. If $u, v \in V$ then $|(u, v)| \cdots\cdots\cdots$.

(A) $= |(v, u)|$ (B) $\leq ||u||||v||$ (C) $= |u||v|$ (D) $= ||u||||v||$

xi. $W^\perp$ is the orthogonal complement of W a subspace of V then $W^\perp$ is a subspace of $\cdots\cdots\cdots$.

(A) W (B) V (C) V/W (D) $V^\perp$

xii. Hom(V, V) is an $\cdots\cdots\cdots$ over F.

(A) algebra (B) field (C) ring (D) module

xiii. If A is an algebra with unit element over F , then A is isomorphic to $\cdots\cdots\cdots$ for some vector space V.

(A) subalgebra of V (B) subalgebra of $A(V)$ (C) subalgebra of A (D) algebra of V

xiv. If V is finite dimensional over F , then $T \in A(V)$ is invertible if and only if the constant term of the $\cdots\cdots\cdots$ for T is not zero.

(A) minimal polynomial (B) characteristic polynomial (C) polynomial in $F[x]$ (D) polynomial in F

xv. If V is finite dimensional over F and if $T \in A(V)$ is invertible then $T-1$ is a polynomial expression in $\cdots\cdots\cdots$ over F.

(A) F (B) T (C) A (D) V

xvi. The relation on $A(V)$ defined by similarity is $\cdots\cdots\cdots$ relation.

(A) partial ordering (B) total ordering (C) equivalence (D) partition

xvii. If $W \subset V$ is invariant under T , then T induces a linear transformation $\overline{T}$ on V/W, then degree of minimal polynomial of T $\cdots\cdots\cdots$ degree of minimal polynomial of $\overline{T}$.

(A) $=$ (B) $\leq$ (C) $\geq$ (D) $\neq$

xviii. If $T \in A(V)$ has all its characteristic roots in F, then there is a basis of V in which the matrix of T is $\cdots\cdots\cdots$.

(A) Triangular (B) Diagonal (C) Identity (D) Jordan

xix. Matrix for any $T \in A(V)$ with respect to two different basis of V are $\cdots\cdots\cdots$.

(A) different (B) same (C) equal (D) inverse of each other

xx. $T \in A(V)$ is nilpotent if for some k $T^k = 0$ but $\cdots\cdots\cdots$.

(A) $T^k \neq 0$ (B) $T^{k+1} \neq 0$ (C) $T^{k-1} = 0$ (D) $T^{k-1} \neq 0$

xxi. If W of dimension m , is cyclic with respect to T , then the dimension of $T^k(W)$ is $\cdots\cdots\cdots$ for all $k \leq m$.

(A) $k - m$ (B) m (C) $m - k$ (D) k

xxii. If the minimal polynomial and characteristic polynomial for $T \in A(V)$ is $(x-1)^2(x+1)^2$ then $\cdots\cdots\cdots$ positions of super diagonal are 1's .

(A) $(1,2),(2,3)$ (B) $(1,2),(3,4)$ (C) $(2,3),(3,4)$ (D) $(1,3),(2,3)$

xxiii. If the minimal polynomial $(x-1)^3$, characteristic polynomial $(x-1)^4$ for $T \in A(V)$ and geometric multiplicity of 1 is 2 then Jordan form for T is $\cdots\cdots\cdots$

(A) $\text{diag} J_3(1), J_1(1)$ (B) $\text{diag} J_2(1), J_2(1)$ (C) $\text{diag} J_1(1), J_3(1)$ (D) $\text{diag} J_4(1)$

xxiv. A polynomial with coefficients which are complex numbers has all its roots in the $\cdots\cdots\cdots$.

(A) real field (B) complex field (C) field of quotients (D) field of quaternary

xxv. The only irreducible , nonconstant polynomial over the field of real numbers are either of degree $\cdots\cdots\cdots$.

(A) 1 or 2 (B) 2 or 3 (C) 1 or 3 (D) 2 or 4

xxvi. A unitary transformation is one which is one which preserves all the $\cdots\cdots\cdots$.

(A) structure of V (B) structure of F (C) roots of T (D) polynomial

xxvii. If T is unitary and if λ is characteristic root of T , then $|\lambda| = \cdots\cdots\cdots$.

(A) 0 (B) 2 (C) 1 (D) i

xxviii. Hermitian over the real field are just $\cdots\cdots\cdots$.

(A) diagonal (B) symmetric (C) skew-symmetric (D) triangular

xxix. The set of all bilinear forms on V is a $\cdots\cdots\cdots$ of the space of all functions from $V \times V$ into F.

(A) superspace (B) subspace (C) field (D) bilinear space

xxx. The space of bilinear forms on V is denoted by $\cdots\cdots\cdots$.

(A) $L(V, F)$ (B) $L(V, V)$ (C) $L(F, F, V)$ (D) $L(V, V, F)$