Seat No.	

PD-11

Total No. of Pages : 10

Ph.D. Entrance Examination, 2025
MATHEMATICS
Subject Code : 58796

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Day and Date : Wednesday, 10-09-2025 **Total Marks : 100**

Time : 04.00 p.m. to 6.00 p.m.

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Instructions:

1. All questions are compulsory.
2. Each question carries 2 marks.
3. Answers should be marked in the given OMR answer sheet by darkening the appropriate option.
4. Use black ball point pen only for marking the circle. Do not make any stray mark on the OMR Answer Sheet.
5. Follow the instructions given on OMR Sheet.
6. Rough work shall be done on the sheet provided at the end of question paper.
7. Only non programmable calculators are allowed.

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Choose the correct answer

1)	What does the term "sampling" refer to in research? A) Collecting data B) Selecting a subset of the population C) Analyzing data D) Presenting findings
2)	What is the initial step in initiating the research process? A) Exploring sources of information to identify a problem. B) Reviewing related literature. C) Identifying the problem. D) Exploring solutions to the problem.
3)	How can the depth of a research study be assessed? A) Based on the research title B) Considering the total expenditure on research C) Examining the duration of the research D) Evaluating the research objectives
4)	Which of the following is not categorized as a "Graphic representation"? A) Pie Chart B) Bar Chart C) Table D) Histogram
5)	Which of the following is the first step in starting the research process? A) Searching sources of information to locate the problem. B) Survey of related literature C) Identification of the problem D) Searching for solutions to the problem

6)	Research can be classified as: A) Basic, Applied and Action Research B) Philosophical, Historical, Survey and Experimental Research C) Quantitative and Qualitative Research D) All the above
7)	Which of the following is the Objective of the Research? (A) To become familiar with a phenomenon (B) To test a hypothesis of a causal relationship between variables (C) To determine the frequency with which something occurs or with it is associated with something else. (D) All of the above
8)	Arrange the following words in a meaningful sequence - 1. Sun 2. Rain 3. Child 4. Rainbow 5. Happy (A) 2, 1, 4, 3, 5 (B) 3, 2, 1, 4, 5 (C) 2, 1, 3, 4, 5 (D) 4, 5, 1, 3, 2
9)	Which of the following plagiarism software is available under open access? (A) Turnitin (B) Urkund **(C) Viper** (D) All of these invalid

10) In the question below are given four statements followed by four conclusions numbered I, II, III and IV. Read all the conclusions and then decide which of the given conclusion logically follows from the given statement.

Statements: All shoes are tables. Some tables are lanes. All caps are lanes. Some lanes are row.

Conclusions: I. Some tables are rows. II. Some tables are shoes. III. Some rows are caps. IV. Some lanes are shoes.

(A) Only I and II follow

(B) Only II follows

(C) Only III follows

(D) Only either I or IV follow

(E) None of these

11) What is the rate of discount if a car which price was $4,000 was sold for $3,200 ?

(A) 14%

(B) 16%

(C) 18%

(D) 20%

12) Which of the following is a step of research design?

(A) Defining the problem and formulating a hypothesis

(B) Collecting data

(C) Drawing inferences from the data

(D) All of the above

13) The "em quad" is the unit to measure _____ in a formula.

A) height of a symbol B) font size of a symbol

C) blank space between symbols D) the size of the kernel letter

14) Which of the following is an example of professional writing?

A) For a matrix A, denote its transpose as $\overline{A}$.

B) The radial acceleration of a particle is $r'' - r(\theta')^2$.

C) $\dot{y} + Py = Q$ is a linear differential equation.

D) None of these

15) Which of the following is an example of professional writing?

A) Consider the direct sum $A_1 \oplus A_2 \oplus \ldots A_m$.

B) Let $a_1, a_2 \cdots a_n$ be integers.

C) ..... much English mathematical writing is a result of Ramanujan.

D) Euler's formula is $1 + 1/2^2 + 1/3^2 + \cdots$ to $\infty = \pi^2/6$.

16) Which of the following symbol is known as *existential quantifier*?

A) $\exists$ B) $\therefore$ C) $\forall$ D) $\because$

17) Which of the following is the correct abbreviation of mathematical word?

A) Lim for limit. B) Re for real part.

C) Sin for sine. D) Exp for exponential.

18) _____ is used to identify an error in fact, or grammer, or spelling that has occured in the original.

A) [*sic*] B) *i. e.* C) [*scil.*] D) [*q.v.*]

19) How do you write x_m^n in LaTeX?

(a) `$x_m^n$`

(b) `$x^m_n$`

(c) `$^n_x_m$`

(d) `$m^n_x$`

20) The LaTeX command

```
$\sqrt[4]{5}$
```

produces

(a) $4\sqrt{5}$
(b) $\sqrt[5]{4}$
(c) $\sqrt[4]{5}$
(d) $5\sqrt{4}$

21) The latex Command for type style boldface is

(a) `\boldtext{...}`
(b) `\textbf{...}`
(c) `\bftext{...}`
(d) `\bold{...}`

22) The latex command

```
$\ddot{a}$
```

produces

(a) $''a$
(b) a''
(c) $..a$
(d) $\ddot{a}$

23) If O=zeros(2,2) then what is the output of the following command?
size(O,"r")
a) 1 b) 2 c) 3 d) 4

24) In Scilab we use linspace(1,10,5) in order to produce ..... values in the interval [1; 10].
a) 5 b) 50 c) 10 d) 100

25) If the condition is not satisfied, the ..... statement allows to perform an alternative statements.
a) continue b) if c) else d) exit

26) If Lagrangian L is independent of q, then corresponding ---- is constant.
(a) energy (b) linear momentum (c) generalized momentum (d) angular momentum

[5]

27) The extremal of the functional $I = \int_{x_1}^{x_2} x\sqrt{1 + y'^2}\, dx$ is a

a) catenary
b) cycloid
c) arc of the great circle
d) circle

28) Suppose $\{f_n\}$ is a sequence functions, differentiable on [a,b]. If $\{f_n'\}$ converges uniformly to a function g on [a,b] then
(a) the sequence $\{f_n\}$ converges to f on [a,b] , f is differentiable and $f' = g$
(b) the sequence $\{f_n\}$ converges to f uniformly on [a,b] , f is differentiable and $f' = g$
(c) the sequence $\{f_n\}$ converges to f on [a,b] , f is differentiable and $f' = g$ provided the sequence $\{f_n(x_0)\}$ converges for some $x_0 \in [a, b]$
(d) none of the above

29) Let $f = (f1, f2)$ be the mappings given by $1(x, y) = e^x \cos y$, $f2(x, y) = e^x \sin y$. The Jacobian of the function f is
(a) 0 (b) 1 (c) e^x (d) e^{2x}

30) Consider the real line $\mathbb{R}$, $\mathcal{T} = P(\mathbb{R})$ and $\mathcal{T}'$ = standard topology on $\mathbb{R}$. Then,

a) only $(\mathbb{R}, \mathcal{T}')$ is metrizable
b) neither $(\mathbb{R}, \mathcal{T})$ nor $(\mathbb{R}, \mathcal{T}')$ are metrizable
c) only $(\mathbb{R}, \mathcal{T})$ is metrizable
d) $(\mathbb{R}, \mathcal{T})$ and $(\mathbb{R}, \mathcal{T}')$ both are metrizable

31) Consider the set $\mathbb{R}$ with standard topology on it, and let $A = \{\frac{1}{n} : n \in \mathbb{Z}_+\}$. Then $\bar{A} =$

a) A b) $A \cup \{0\}$
c) $A \cup (0, 1)$ d) $A \cup [0, 1]$

32) Let γ be a line segment $[1, i]$ and $f(z) = |z|^2, z \in \mathbb{C}$. Then $\int_\gamma f =$

(a) $1 + i$
(b) $1 - i$
(c) $\frac{2}{3}(1 + i)$
(d) $\frac{2}{3}(1 - i)$

33) The singularity $z = 0$ of $f(z) = \frac{\sin z}{z}$ is

(a) a removable singularity
(b) a pole
(c) an irremovable singularity
(d) an essential singularity

34) For the I.E. $g(s) = f(s) + \lambda \int_0^1 g(t)dt$ the Fredholm determinant $D(\lambda) =$

a) $1-\lambda$ b) $1+\lambda$ c) $1+\frac{\lambda}{4}$ d) $1-\frac{\lambda}{4}$

35) The third iterated kernel $k_3(s,t)$ of the kernel k(s,t)=1 ,$(s,t) \in [0,1] \times [0,1]$ is

a) e^{s-1} b) e^{s+1} c)1 d) $(st)^3$

36) Let S, S_1, S_2 be any subsets of Hilbert space H. Consider the statements

I) $S_1 \subseteq S_2 \Rightarrow S_2^{\perp} \supseteq S_1^{\perp}$.

II) $S \subseteq S^{\perp\perp}$.

Then

a) Only I is true
b) Only II is true
c) both I and II are true
d) both I and II are false.

37) Let N be a Banach space and N^* is its conjugate space.

Consider the statements

I) If N^* is separable $\Rightarrow$ N is separable.

II) N^* is reflexive $\Leftrightarrow$ N is reflexive.

Then

a) Only I is true
b) Only II is true
c) both I and II are true
d) both I and II are false.

38) If $A = (3,9)$ then $m^*(A)$ is

a) 0 b) 12 c) 3 d) 6

39) Let f and g be two nonnegative functions and $f, g \in L^p$, $0 < p < 1$. Then

a) $\|f+g\|_p \le \|f\|_p + \|g\|_p$ b) $\|f+g\|_p \le \|f\|_p \cdot \|g\|_p$

c) $\|f+g\|_p \ge \|f\|_p + \|g\|_p$ d) $\|f+g\|_p \ge \|f\|_p \cdot \|g\|_p$

40) Consider a group $G = S_3 = \{(1), (1\ 2), (1\ 3), (2\ 3), (1\ 2\ 3), (1\ 3\ 2)\}$ and the set $X = \{1,2,3\}$.

The isotropy subgroup $G_2 =$_____.

A) G B) $\{(1), (1\ 2\ 3)\}$
C) $\{(1), (1\ 3)\}$ D) $\{(1)\}$

41) Consider the following statements:

I) $x+1$ is a factor of x^2+1 in $Z_2[x]$.

II) $2x^5 - 5x^4 + 5$ is reducible over $\mathbb{Q}$.

A) Both I) and II) are true B) Both I) and II) are false
C) Only II) is true D) Only I) is true

42) A splitting field of $f(x) = x^2 - 1 \in \mathbb{R}[x]$ over $\mathbb{R}$ is _____.

A) $\mathbb{R}$ B) $\mathbb{Q}$

C) $\mathbb{C}$ D) $\mathbb{Q}(i)$

43) $[\mathbb{Q}(2^{1/4}, i): \mathbb{Q}(2^{1/4})] =$ _____.

A) 4 B) 2

C) 8 D) i

44) If $A = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$, then which of the following is A^{-1}?

A) $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ B) $\begin{bmatrix} -\cos\theta & \sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$

C) $\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$ D) $\begin{bmatrix} \cos\theta & \sin\theta \\ \sin\theta & -\cos\theta \end{bmatrix}$

45) If $\boldsymbol{x}^{(k+1)} = \boldsymbol{H}\boldsymbol{x}^{(k)} + \boldsymbol{c}, k = 0, 1, 2, \ldots$ is the Gauss-Seidal iteration method used to solve the system of n linear algebraic equations $\boldsymbol{A}\boldsymbol{x} = \boldsymbol{b}$, then $\boldsymbol{c}$ =_____.

A) $(\boldsymbol{D} + \boldsymbol{L})^{-1}\boldsymbol{b}$ B) $\boldsymbol{D}^{-1}\boldsymbol{b}$

C) $-\boldsymbol{D}^{-1}(\boldsymbol{L} + \boldsymbol{U})$ D) $-(\boldsymbol{D} + \boldsymbol{L})^{-1}\boldsymbol{U}$

46) If $P_n(x)$ be the n-th Legendre polynomial, then $P_n(1)$ =_____?

(A) ∞

(B) n

(C) 0

(D) 1

47) Which of the following is the indicial polynomial $q(r)$ for the Euler equation $x^2y'' + xy' - 4y = 0$ $(x > 0)$?

(A) $-r^2 - \mathrm{r} + 4$

(B) $r^2 + \mathrm{r} - 4$

(C) $r^2 - 4$

(D) $r^2 + 1$

48) Let W_1, W_2 be subspaces of V a finite dimensional vector space over F. $A(W_1 + W_2) =$ ..... .

(A) $A(W_1 \bigcup W_2)$ (B) $A(W_1 \bigcap W_2)$ (C) $A(W_1) \bigcap A(W_2)$ (D) $A(W_1) \bigcup A(W_2)$

49)	The normal transformation is hermitian if and only if its characteristic roots are ·····. (A) real (B) complex (C) zero (D) absolute value 1
50)	····· ways are there to pick first a vowel and then a consonant from BOAT. (A) 4 (B) 8 (C) 6 (D) 2

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- Rough Work -