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| No. | | |

Total No. of Pages : 13

Ph.D. Entrance Examination 2024-25

MATHEMATICS

Subject Code : 58796

| Day and Date : Wednesday, 13/11/2024 | Total Marks : 100 |
|--------------------------------------|--------------------------|
| Time : 04.00 pm to 06.00 pm | |

Instruction :

- 1) All questions are compulsory.
- 2) Each question carries 2 marks.
- 3) Answers should be marked in the given OMR answer sheet by darkening the appropriate option.
- 4) Use black ball point pen only for marking the circle. Do not make any stray mark on the OMR Answer Sheet.
- 5) Follow the instructions given on OMR Sheet.
- 6) Rough work shall be done on the sheet provided at the end of question paper.
- 7) Only non-programmable calculators are allowed.

Choose the correct answer

- 1) What does the torin "peer-reviewed journal" mean?
 - A) A journal edited by colleagues
 - C) A journal with no editorial board
 - B) A journal that only publishes positive findings
 - D) A journal with biased reviews

- 2) Research is
 - A) Searching again and again
 - B) Finding solution to any problem
 - C) Working in a scientific way to search for truth
 - D) None of the Above
- 3) In which of the following does the format of thesis writing closely reserable?
 - A) Writing of Seminar representation
 - B) Preparation of research paper/article
 - C) A research dissertation
 - D) Presenting a workshop/conference paper
- 4) Why do you need to review the existing literature?
 - A) To make sure you have a long list of references
 - B) Because without it, you could never reach the required word-count
 - C) To find out what is already known about your area of interest
 - D) To help in your general studying
- 5) The authors other than the corresponding author are listed in order of their the research project.
 - A) Age
 - B) Soniority
 - C) Contributions
 - D) Affection

- 6) Bibliography given in a research report:
 - A) shows vast knowledge of the researcher
 - B) helps those interested in further research
 - C) has no relevance to research
 - D) all the above
- 7) Research is basically
 - A) a methodology of enquiry
 - B) search of truth
 - C) a systematic exploration of facts
 - D) All of the above
- 8) If AIRLINE is written as ENILRIA7, then RAILWAY will be written as -
 - A) YAWLIAR7
 - B) YAWLIAR8
 - C) YAWILAR7
 - D) YAWILAR8
- 9) Which of the following acts constitute plagiarism?
 - A) Presenting other's work as own
 - B) Paraphrasing without citation
 - C) Quoting other's work without quotation marks
 - D) All of these

| 10) | Tuberculosis: Lungs:: Cataract:? | |
|-----|--|--------------------------|
| | A) Ear | B) Throat |
| | C) Skin | D) Eye |
| 11) | -4 + 4 - 4+4 = ? | |
| | A) 0 | B) 2 |
| | C) 4 | D) 8 |
| 12) | The issue of 'research ethics' may be considered pertinent at which st of research? | |
| | A) At the stage of problem formula | ation and its definition |
| | B) AT the stage of defining the pop | oulation of research |
| | C) AT the stage of data collection a | and interpretation |
| | D) At the stage of reporting the fin | dings |
| 13) | Which of the following are frozen s | ymbols? |
| | e, A, ∏, ∈, x , ∝ | |
| | A) e, A, ∏ | B) ∏, ∈, x |
| | C) ∏, ∈, ∝ | D) e, ∏, ∈ |
| 14) | The correct abbreviation for limit is | 5 |
| | A) lim | B) lim |
| | C) Lim | D) Lt. |
| 15) | In the kernel-index notation, if the size of the kernel letter is an <i>m</i> -pt font then the size of the index letter is | |
| | A) (m-2)-pt | B) (m+2)- pt |
| | C) 2m – pt | D) (m-1)-pt |

- 16) Which of the following is/are Principle(s) of professional writing?
 - i) Abbreviate units of measurement when used with numerical values.
 - ii) Use the same abbreviation (of an unit of measurement) for the singular and the plural.
 - A) only (i) B) only (ii)
 - C) both (i) and (ii) D) none of (i) and (ii)
- 17) Which of the following is an example of professional writing?
 - A) This is a Toeplitz matrix.
 - B) Newton is unstable.
 - C) This matrix is a Toeplitz.
 - D) This matrix is the Toeplitz.
- 18) Which of the following is an example of professional writing?
 - A) Let X is a set.
 - B) Let *f*, *g* are functions.
 - C) Let 'e' be the exponential number.
 - D) Let 'e' denote the exponential number.
- 19) How do you write \sqrt{x} in LaTeX?
 - A) $sqr{x}$
 - B) ${1/2} root{x}$
 - C) ${2}\root{x}$
 - D) $\operatorname{sqrt}{x}$

- 20) What LaTeX command is used for the infinity symbol?
 - A) \$\infty\$
 - B) \$\infinity\$
 - C) \$\infinite\$
 - D) \$\inf\$
- 21) How do you write *a* in LaTeX?
 - A) $dot{a}$
 - B) \${a}\dots
 - C) \${\cdot}^{a}\$
 - D) \${a}^{\cdot}\$
- 22) To write $\begin{pmatrix} u & v \\ w & x \end{pmatrix}$ the Latex command is

 - B) $\begin{pmatrix} u \& v \setminus w \& x \end{pmatrix}$
 - C) $\begin{vmatrix} u & v \ w & x \end{vmatrix}$
 - D) $\begin{Vmatrix} u \& v \setminus w \& x \end{matrix}$
- 23) The statement i=5:1:8 produces i.....
 - A) 5. 6. 7. 8. B) 5. 6. 7.
 - C) 1.2.3. D) 3.
- 24) To configure the ttitle of our plot, we use thefunction.
 - A) xtitle B) title
 - C) X axis D) Y axis

- 25) We use..... statement in loops where, if some condition is satisfied, the loops should not be continued further.
 - A) continue B) break
 - C) end D) exit
- 26) If {f} and { g_n } converges uniformly on some set E then { $f_n g_n$ } converges uniformly on E if
 - A) $\{f_n\}$ or $\{g_n\}$ is sequence of bounded functions
 - B) $\{f_n\}$ and $\{g_n\}$ are sequences of bounded functions
 - C) $\{f_n\}$ or $\{g_n\}$ are sequences of continuous functions
 - D) $\{f_n\}$ and $\{g_n\}$ are sequences of continuous functions

27)
$$\int_{a}^{b} \lim_{n\to\infty} f_n(x) dx = \lim_{n\to\infty} \int_{a}^{b} f_n(x) dx \text{ if}$$

- A) The sequence $\{f_n\}$ is a sequence of Riemann integrable functions and $f_n \rightarrow f$
- B) $f_n \rightarrow f$ uniformly
- C) $f_n \rightarrow f$ uniformly and each f_n is Riemann integrable.
- D) The sequence {f} is a sequence of Riemann integrable functions, $f_n \rightarrow f$ and f is Riemann integrable.
- 28) If the constraint relations are independent of velocity then constraints are classified as
 - A) Scleronomic
 - B) rheonomic.
 - C) holonomic
 - D) dissipative

| 29) | If the Hamiltonian of the dynamical system is given by $H = -pq$ then as |
|-----|--|
| | $t \rightarrow \infty$ |

| A) $q \rightarrow \infty, p \rightarrow 0$ | B) $q \rightarrow 0, p \rightarrow 0$ |
|---|--|
| C) $q \rightarrow \infty, p \rightarrow \infty$ | D) $q \rightarrow 0, p \rightarrow \infty$ |

- 30) Basis element of *k*-topolgy of \mathbb{R} is
 - A) [a, b] B) [a, b)
 - C) (a, b] D) (a, b)
- 31) Consider the following two statements:
 - A) Every second countable space is first countable space
 - B) Every first countable space is second countable space Then,
 - A) only (I) is true B) only (II) is true
 - C) both (I) and (II) are true D) both (I) and (II) are false

32) The harmonic conjugate of $u(x, y) = x^2 - y^2, x, y \in \mathbb{R}$ is A) $x^2 + y^2$ B) 4xy

C) 2xy D) $x^2 - y$

33) $\Upsilon(t) = a + r e^{it}, t \in [0, 2\pi] \text{ then } \int_{\Upsilon} \frac{dz}{z - a} =$ A) $2\pi i$ B) πi C) 0D) $\frac{\pi i}{2}$

34) The solution of integral equation $y(s) = \frac{5}{6}s - \frac{1}{9} + \frac{1}{3}\int_{0}^{1} (s+t)y(t)dt$ is A) y(s)=0 B) $y(s) = s^{2}$

C)
$$y(s) = s$$
 D) $y(s) = s^{3}$

- 35) Eigen functions g(s) and $\Psi(x)$ corresponding to distinct eigen values of the homogenous integral equations and it's transpose are......
 - A) Orthogonal B) Linearly Independent
 - C) Orthonormal D) Linearly dependent
- 36) Let N be normed linear space and N* is its conjugate space.

Consider the statements

- I) If N* is separable \Rightarrow N is separable.
- II) N* is reflexive N \Leftrightarrow is reflexive.
- A) Only I is true B) Only II is true
- C) both I and II are true D) both I and II are false.
- 37) Consider the statements
 - I) If T is positive operator on Hilbert space H then 1+T is non-singular.
 - II) If T is an operator on Hilbert space H then *I*+*T***T* and *I*+*TT** are Non-singular.

Then

- A) Only I is true
- B) Only II is true
- C) both I and II are true
- D) both I and II are false.

38) If
$$A = \left\{ \frac{1}{n} : n \in \mathbb{N} \right\}$$
 is
A) 0 B) $\frac{1}{n}$
C) ∞ D) 1

39) f' = g a.e. if f and g have same domain and

- A) $m{x: f'(x) = g(x)} \neq 0$
- B) $m{x: f'(x) \neq g(x)} \neq 0$
- C) $m{x: f'(x) = g(x)} = 0$
- D) $m{x: f'(x) = g(x)} = 0$

40) A Sylow 3-subgroup of a group of order 54 has order

| A) | 9 | B) 6 |
|----|----|------|
| C) | 27 | D) 3 |

41) Let *p* be a prime number. Consider the following statements:

- I) Any group of order p^2 is abelian.
- II) If |G| = p then G' = {e}.
- A) Both I) and II) are true
- B) Both I) and II) are false
- C) Only II) is true
- D) Only I) is true
- 42) If *p* and *q* be distinct prime numbers, then is the basis of $\mathbb{Q}(\sqrt{p},\sqrt{q})$ over \mathbb{Q} ?
 - A) {1, p, q} B) {1, p, q, pq} C) { $1, \sqrt{p}, \sqrt{q}, \sqrt{p}\sqrt{q}$ } D) { $\sqrt{p}, \sqrt{q}, \sqrt{p}\sqrt{q}$ }
- 43) $\mathbb{Q}(\sqrt{-2})$ is extension of \mathbb{Q} .
 - A) finite but not normal B) finite and normal
 - C) normal but not finite D) neither finite nor normal

44) If *N* is a positive real number, then $X_{k+1} = \dots, k=0, 1, 2,\dots$ is the iterative method based on the Newton-Raphson method for finding \sqrt{N} .

A)
$$\frac{1}{2} \left(X_k - \frac{N}{X_k} \right)$$

B) $\frac{1}{2} \left(X_k + \frac{N}{X_k} \right)$
C) $X_k \left(2 - NX_k \right)$
D) $\frac{1}{2} \left(X_k - NX_k \right)$

Let A be a square matrix. Consider the following statements:

45)

I) p(A) > ||A||.

- II) If ||A|| 1 then $\lim_{m \to \infty} A^m = 0$.
- A) Both I) and II) are true
- B) Both I) and II) are false
- C) Only II) is true
- D) Only I) is true
- 46) is the solution of the initial value problem $y'' + \frac{1}{x}y' \frac{1}{x^2}y = 0, y(1) = 0$ y'(1) = 1 for x > 0?

A)
$$\phi(x) = x - \frac{1}{x}$$

B)
$$\phi(x) = \frac{1}{2} \left(x + \frac{1}{x} \right)$$

C)
$$\phi(x) = \frac{1}{2} \left(x - \frac{1}{x} \right)$$

D)
$$\phi(x) = x + \frac{1}{x}$$

47) Which of the following is the second successive approximation $\phi_1(x)$ to the solution of y' = 1+xy, y(0) = 1?

A)
$$1-x-\frac{x^3}{3}$$

B) $1+x+\frac{x^3}{3}$
C) $1-x+\frac{x^2}{2}$
D) $1+x+\frac{x^2}{2}$

48) If V is of dimensions *m*, over F, then Hom(V, F) is of dimension over F

| A) | 1 | B) F |
|----|---|-------------|
| C) | V | D) <i>m</i> |

49) A unitary transformation is one which is one which preserves all the.....

- A) structure of V B) structure of F
- C) roots of T D) polynomial

50) Any tree with at least two vertices has more than one vertwx of degree......

- A) 2 B) 1
- C) 3 D) 4

-- ROUGH WORK --