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No.		

P.G. Entrance Examination, 2024

M. Sc. MATHEMATICS

Subject Code : 58716

Day and Date : Thursday, 16-05-2024
Time : 01:00 pm to 02:30 pm

Total Marks : 100

Instructions :

- 1) All questions are compulsory.
- 2) Each question carries 1 mark.
- 3) Answers should be marked in the given OMR answer sheet by darkening the appropriate option.
- 4) Follow the instructions given on OMR sheet.
- 5) Rough work shall be done on the sheet provided at the end of question paper.
- 1. If f and g are bounded functions defined on [a, b] and P is any partition of [a, b], then which of the following is true?
 - A) $U(f+g, P) \leq U(f, P) + U(g, P)$
 - B) $U(f+g, P) \ge U(f, P) + U(g, P)$
 - C) L(f + g, P) = L(f, P) + L(g, P)
 - D) $L(f+g, P) \leq L(f, P) + L(g, P)$
- 2. Let f(x) = x on [0, 1] and $P = \{0, \frac{1}{3}, \frac{2}{3}, 1\}$ be a partition of [0, 1], then U(f, P) =_____
 - A) $\frac{1}{3}$ B) 1 C) $\frac{2}{3}$ D) $\frac{3}{4}$

If g is a continuous function on [a, b] that is differentiable on (a, b) and if g' is integrable on [a, b] then ∫_a^b g' = ____.

A) g(x) B) g'(b) - g'(a) C) g(a) - g(b) D) g(b) - g(a)

4. If $f: [0, 1] \rightarrow \mathbb{R}$ such that $f(x) = \begin{cases} 1 & \text{if } x \text{ is rational} \\ -1 & \text{if } x \text{ is irrational} \end{cases}, \text{ then } ____.$ A) f and |f| both integrable B) f is integrable but |f| is not integrable C) Neither f nor |f| integrable D) |f| is integrable but f is not integrable 5. $\lim_{x\to 0} \frac{1}{x} \int_0^x e^{t^2} dt =$ _____. A) e^{x^2} D) 2 *e*^{x²} B) 0 C) 1 6. The improper integral $\int_a^b \frac{dx}{(x-a)^n} dx$ is converges if _____. A) n > 1B) n = 1C) $n \neq 1$ D) *n* < 1 7. The Cauchy Principal Value of $\int_{-\infty}^{\infty} \sin t \, dt$ is _____. C) $\frac{1}{2}$ B) 0 A) 1 D) Does not exist

8. The value of the Fourier coefficient a_0 in the Fourier series

9. If f(x) is expanded in a series of sines of the form $f(x) = \sum_{n=1}^{\infty} b_n \sin nx$, then $b_n =$

(A) $\frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx \, dx$ (B) $\frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx \, dx$ (C) $\frac{2}{\pi} \int_{0}^{\pi} f(x) \cos nx \, dx$ (D) $\frac{2}{\pi} \int_{0}^{\pi} f(x) \sin nx \, dx$

10. In the Fourier series expansion of $f(x) = x \sin x$ in $[-\pi, \pi]$, the value of the Fourier coefficient b_n is _____.

A) 2 B) 0 C)
$$\frac{-2 \times (-1)^n}{n^2 - 1}$$
 D) $\frac{(-1)^n}{n^2 - 1}$

11. If $T: R^2 \to R^2$ and $S: R^2 \to R^3$ defined by T(x, y) = (y, x) and S(x, y) = (x - y, x + y, y)then ST (x, y) =A) (y + x, y - x, x) B) (y - x, x + y, x) C) (y - x, x + y, y) D) (y + 2x, y - x, x)

12. If u = (1,2,3,4) then norm of u with respect to Euclidean inner product in \mathbb{R}^4 is A) 30 B) 26 C) $\sqrt{30}$ D) $\sqrt{26}$

13. If $\lambda = 1/5$ is an eigen value of an invertible operator T then eigen value of T^{-1} is A) -5 B) 1/5 C) 5 D) -1/5

14. A linear transformation $T: V \rightarrow W$ is non singular if

A) T is not one- one	B) T is not onto
C) Ker $T = \{0\}$	D) Range $T = \{0\}$

15. The eigen values of the matrix $\begin{bmatrix} -1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 2 \end{bmatrix}$ areA) 1,2,3B) -1,1,0C) 3,2,1D) -1,2,1

16. Let $T: \mathbb{R}^2 \to \mathbb{R}^2$ be a linear transformation and $\{e_1 = (1,0), e_2 = (0,1)\}$ be the standard basis of \mathbb{R}^2 . If $T(e_1) = (1,2)$ and $T(e_2) = (2,3)$ then T(x,y) =

A) (2x + y, 2x + 3y) B) (2x, 3y) C) (x + 2y, 2x + 3y) D) (x + y, x - y)

17. If $T: \mathbb{R}^2 \to \mathbb{R}^2$ defined by T(x, y) = (x - y, x) is an invertible linear transformation, then $T^{-1}(a, b) =$ A) (a, a - b) B) (b, a + b) C) (b - a, -b) D) (b - a, b)

18. The characteristic polynomial of the matrix $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ is A) $x^2 - 5x - 2$ B) $x^2 + 5x - 10$ C) $x^2 - 3x$ D) $x^2 - 3x - 10$

19. The set $S = \{(1,2), (3,4), (5,6)\}$ of vectors in \mathbb{R}^2 is

A) a linearly independent subset B) a basis of R^3

C) a linearly dependent subset D) an orthogonal set

20. If
$$u = (1,1,1)$$
 and $v = (1,2,0)$ then $|| u + v || =$
A) $\sqrt{14}$ B) 14 C) $\sqrt{7}$ D) 3

- 21. For solving the equation $\frac{d^2y}{dx^2} + P\frac{dy}{dx} + Qy = R$ by using the method of variation of parameters, we assume the solution in the form y = Au + Bv, where A and B are______A)constants B) functions of y C) functions of x D) functions of x and y.
- 22. The partial differential equation formed by eliminating the constants form z = ax + byA) z = p + q B) z = px + qy C) z = py + qx D) z = ap + bq23. P.I. of $\frac{1}{D^2+a^2} cosax$ is _____. A) $\frac{x}{2a}sinax$ B) $\frac{-x^2}{2!}\frac{1}{4a^2}sinax$ C) $\frac{x}{2a}cosax$ D) $\frac{-x^2}{2!}\frac{1}{4a^2}cosax$ 24. The complementary function (C.F.) of the equation $x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} - 4y = x^2$ is______. A) $y = c_1 x + c_2 x^2$ B) $y = c_1 x^2 + \frac{c_2}{x^2}$ C) $y = \frac{c_1}{x} + c_2 x^2$ D) $y = c_1 x + \frac{c_2}{x}$ 25. The equation $(x^2 - ay)dx + (y^2 - ax)dy = 0$ is______ B) Variable separable D) Linear A) Homogeneous C) Exact 26. $\lim_{x\to 0} x^x$ is_____. **A)**∞ B)1 **C)**0 D)-∞ 27. The value of $\left(\sin\frac{\pi}{3} + i\cos\frac{\pi}{3}\right)^6$ is _____. A)1 B) -1 C)0 D) i 28. If $u = \sin^{-1}\left(\frac{x^2 + y^2}{x + y}\right)$ then $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} =$ ______ A)u B)sin u C) tan u D)sec u 29. If $y = \cos 3x$ then $y_{10} =$ _____. B) $3^{10} \cdot \cos(3x + 5\pi)$ C) $\cos 10x$ D) $3^{10} \cdot \cos 3x$ A) 3¹⁰ 30. The solution of the equation tan(y - px) = p is_____. A) $y = cx - tan^{-1}c$ B) $y = cx + tan^{-1}c$ C) y = cx + tan c D) $y = cx + (tan^{-1}c)^2$

31. If A is Hermitian matrix then iA is ...

A) Symmetric matrix	B) Skew-Hermitian matrix
C) Hermitian matrix	D) Skew Symmetric matrix

32. Let A = $\{1, 2, 3\}$ and R = $\{(1, 3), (3, 1), (2, 3)\}$ be a relation on A, then R is...

A)Symmetric only B) anti-symmetric only C) symmetric as well as anti-symmetric D) neither symmetric nor anti-symmetric 33. If $a \in G$ and $a^m = e$ for some positive integer m, then... B) $O(a) \le m$ C) $O(a) \ge m$ D) $O(a) = \frac{m}{2}$ A) $O(a) \neq m$ 34. Let Z be the set of integers and H=5Z is the subgroup of Z then $[Z: 5Z] = \dots$ A) 10 B) 5 C) 1 5 D) 20 35. An onto homomorphism is called _____ A)isomorphism B) monomorphism C) endomorphism D) epimorphism 36. The set $\mathbb{Q} \cap [0,1]$ is ______ A) countable B) uncountable C) finite D) none of these 37. The least upper bound of the set $\left\{\begin{array}{c} \frac{1}{n} / n \in N\right\}$ is ... A)1 B) 0 C) -1 A)1 D) 2 38. If $\{S_n\} = \{(-1)^n\}_{n=1}^{n=\infty}$, then $\lim_{n \to \infty} SupS_n = \cdots$ A)0 C) -1 D) 2 39. The series $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n^p}$ converges for _____ A. *P* < 0 B. *P* > 0 C. P = 0D. P = -140. If $\{S_n\} = \{sin(\frac{n\pi}{3})\}_{n=1}^{\infty}$, then $\lim_{n \to \infty} inf S_n =$ _____ A. $\sqrt{3}/2$ B. $\sqrt{3/2}$ C. $-\sqrt{3/2}$ D. $-\sqrt{3}/2$

41. A set of non-negative individual allocations ($x_{ij} \ge 0$) which simultaneously removes deficiencies is called

- A) optimal solution B) feasible solution
- C) basic feasible solution D) none of the above

42. The IBFS to the following transportation problem



46. There are five jobs, each of which must go through the two machines A and B in the order AB. Processing times (hours) are given below:

Job	1	2	3	4	5	
Time for A	5	1	9	3	10	
Time for B	2	6	7	8	4	
the total ideal time for machine A is						

the total ideal time for machine A is

A)2 B)3 C)4 D)5

.,

47. A maximum element among the minimum elements selected for each row is called

- A) value of the game B) saddle point
- C) maximin D) minimax
- 48. For the following game

		B ₁	B ₂	B ₃	B ₄
	A_1	8	1	9	5
Player A	A_2	6	5	7	17
	A ₃	7	2	-4	10

Player B

The value of the game is

A)4	B)3	C)6	D)5
/ ·	-)-	-)-	-)-

49. Successful applications of the linear programming techniques may be found in

- A) all of the preceding B) chemical and oil
- C) food processing D) the iron and steel industry

50. For the following game

Player B

		B ₁	B ₂
Player A	A_1	8	1
	A ₂	6	5

The value of game is

A)51/7 B)52/7 C)53/7 D)54/7

- 51. Let G = (V, E) be a graph, where $V = \{u_1, u_2, u_3, u_4, u_5\}$ and
 - $E = \{(u_1, u_1), (u_2, u_3), (u_3, u_1), (u_4, u_2), (u_5, u_2), (u_5, u_1), (u_4, u_2)\}$ then the degree of the vertex u_1 is
 - A) 3 B) 4 C) 5 D) 2

52. Let G be a simple graph. Which of the following statements is true?

- P: Adjacency matrix is symmetric.
- Q: Trace of adjacency matrix is 0.
- A) P only B) Q only C) Both P and Q D) Neither P nor Q

- 53. Consider the following two statements:
 - I: A graph G has Euler Circuit.
- II: A graph G has Hamiltonian Circuit.

Then

- A) Only $(I) \rightarrow (II)$ B) Only $(II) \rightarrow (I)$ C) $(I) \leftrightarrow (II)$ D) Neither $(I) \rightarrow (II)$ nor $(II) \rightarrow (I)$
- 54. The number of spanning trees for the following graph are



A \ 2	\mathbf{D}) \mathbf{A}	(1)	\mathbf{D}) \mathbf{c}
A) 3	B) 4	C) 2	D) 5
		<i>e)</i> -	2)0

55. The minimum number of edges that must be added to an acyclic graph with k components to obtain a tree are

A) k-1 B) k+1 C) k D) k+2

- 56. The compound propositions p and q are called logically equivalent if ______ is a tautology. A) $\neg p \lor \neg q$ B) $p \rightarrow q$ C) $\neg (p \lor q)$ D) $p \leftrightarrow q$
- 57. The statement (~ p ↔ q) ∧ ~q is true if
 A) p: False q: False B) p: True q: True C) p: False q: True D) p: True q: False
- 58. What is the decimal equivalent of the hexadecimal number BF9 ?A) 2802B) 3065C) 2048D) 1024
- 59. Subtraction of the binary numbers 11000 and 1011 is
 A) 1101
 B) 1110
 C) 1100
 D) 1010
- 60. Suppose x is a real number. Let p: 0 < x, q: x < 3, and r: x = 3. Then the inequality $0 < x \le 3$ can be written symbolically as:
 - A) $p\Lambda(qVr)$ B) $p\Lambda r$ C) pVqVr D) $p\Lambda q\Lambda r$
- 61. $L\{\sin 2t \cos 2t\} =$ _____.

A)
$$\frac{2}{s^2-16}$$
 B) $\frac{2}{s^2+16}$ C) $\frac{2s}{s^2+16}$ D) $\frac{2s}{s^2-16}$

62.
$$L\{t \cos 3t\} =$$
 _____.
A) $\frac{s^2 - 9}{(s^2 + 9)^2}$ B) $\frac{s}{s^2 + 9}$ C) $\frac{3}{s^2 + 9}$ D) $\frac{-6s}{(s^2 + 9)^2}$

63. If F(t + 2) is a Heaviside unit step function then $L\{F(t + 2)\} =$ _____.

A)
$$e^{-2s}$$
 B) $\frac{e^{2s}}{s}$ C) $\frac{e^{-2s}}{s}$ D) e^{2s}
64. $L^{-1}\left\{\frac{1}{4s-5}\right\} =$.
A) $\frac{1}{4}e^{-\frac{5}{4}t}$ B) $\frac{1}{4}e^{\frac{5}{4}t}$ C) $\frac{1}{5}e^{-\frac{5}{4}t}$ D) $\frac{1}{5}e^{\frac{5}{4}t}$
65. If $L^{-1}\{f(s)\} = 3t^2$ then $L^{-1}\left\{\frac{f(s)}{s}\right\} =$.
A) t B) t^2 C) t^3 D) $\frac{t^3}{3}$
66. $L^{-1}\left\{\frac{1}{s^2-16}\right\} = F(t)$
A) $\frac{1}{4}\cosh 4t$ B) $\frac{1}{4}\cos 4t$ C) $\frac{1}{4}\sin 4t$ D) $\frac{1}{4}\sinh 4t$

67. $L^{-1}\left\{\frac{1}{s^2}\right\} =$ _____ A) $\frac{1}{\sqrt{t}}$ B) $\frac{1}{\sqrt{\pi t}}$ C) $\frac{\pi}{\sqrt{t}}$ D) $\sqrt{\frac{\pi}{t}}$

68. If f(s) is Fourier transform of F(X) then Fourier transform of F'(X) is _____ where $F{F(x)} = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} F(x)e^{isx} dx$ A) is f(s) B) - s f(s) C) -is f(s) D) s f(s)69. The inversion formula for the infinite Fourier transform is _____

A) $F(x) = \frac{2}{\pi} \int_0^\infty F(s) \sin sx \, ds$ B) $F(x) = \frac{1}{\pi} \int_0^\infty F(s) \sin x \, ds$ C) $F(x) = \frac{1}{\pi} \int_0^\infty F(s) \sin sx \, ds$ D) $F(x) = \frac{2}{\pi} \int_0^\infty F(s) \sin x \, ds$ 70. If F(x) is continuous and F'(x) is piecewise continuous, then $F_s\{F'(x)\} =$ _____

A) $F_c\{F(x)\}$ B) $-F_c\{F(x)\}$ C) $-s^2F_c\{F(x)\}$ D) $-sF_c\{F(x)\}$ 71. The value of the integral $\int_C \frac{dz}{z^2(z+3)}$ taken counterclockwise around the circle |z| = 2 is

A)
$$\frac{2\pi i}{9}$$
 B) $2\pi i$ C) $-\frac{2\pi i}{9}$ D) $\frac{2\pi i}{3}$

72. The residue of $z^2 e^{1/z}$ at z = 0 is

73. If f(z) = u + iv then Cauchy Riemann equations are

$$A)u_x = v_x , u_y = v_y \quad B)u_x = v_y , u_y = v_x$$
$$C)u_x = v_y , u_y = -v_x \quad D)u_x = -v_y , u_y = v_x$$

74. If f(z) = u + iv is analytic function in finite region and $u = x^3 - 3xy^2$ then $v = \dots$

A)
$$3x^2y + y^3 + c$$
 B) $3x^2y - y^3 + c$ C) $3x^2y - y^2 + c$ D) $3xy + y + c$

75. The analytic function whose real part is e^x (xcosy - ysiny) is.....

A)
$$ze^{x^2} + iy$$
 B) z^2e^z C) $ze^z + ci$ D)None of these

76. A curve without multiple points is called

A) Jordan curve B) arc C) regular curve D) None of these

77. If f(z) = u(x, y) + iv(x, y) is such that both u(x, y) and v(x, y) satisfies Cauchy Riemann equations and if $u_x(x, y) = x$ and $u_y(x, y) = -y$ then for any z = (x, y), the value of $f'(z) = \cdots$

A) z B) -z C) 0 D) \bar{z}

78.. If $f(z) = |z|^2$ then f is A) differentiable B) nowhere differentiable C) differentiable only at z = 0D) analytic at z = 0

79. If $f(z) = \sqrt{|xy|}$ then A) f is differentiable at z = 0B) C-R equations are not satisfied at 0. C) f is not differentiable at z = 0D) f is nowhere continuous

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80. If $u(x, y) = x^2 - y^2$ then corresponding analytic function $f(z) =$							
L	A) $z^2 + c$ B) $z^3 + c$ C) z	$+c$ D) z^2	3 + ic			
81.	81. Every Cauchy sequence in discrete metric space is						
	A) divergent	B) convergent	C) oscillatory	D) none of these			
82.	82. Which of the following is an open subset of an absolute metric space R?						
	A) {a}	B) (a, ∞)	C) (a, b]	D) none of these			
83. In	any metric space arbi	itrary union of open se	ts				
	A) need not be ope	en set	B) is closed set				
	C) is open set		D) neither open set	nor closed set			
84.	If M is the clo	osed interval [0,1]	with absolute valu	the metric, then B $\left[\frac{1}{4};\frac{1}{2}\right]$			
		B) $\left(0,\frac{3}{4}\right)$	C) $\left(\frac{1}{4}, \frac{3}{4}\right)$	D) $[0, \frac{3}{4}]$			
85.	Let E be a subset of	the metric space M. T	hen a point $x \in M$ is ca	alled Limit point			
	of E if						
	A) there exists a seq	uence $\{x_n\}_{n=1}^{\infty}$ of poin	ts of M which diverge	s to x			
	B) there exists a seq	uence $\{x_n\}_{n=1}^{\infty}$ of point	ts of E which converg	es to x			
	C) there exists a seq	uence $\{x_n\}_{n=1}^{\infty}$ of point	ts of E which diverges	to x			
D) there exists a sequence $\{x_n\}_{n=1}^{\infty}$ of points of M which converges to x							
86. Let f be a function from a set A into a metric space M. Then the function f is							
	bounded if						
	A) f(A) is bounded	subset of M	B) A is bounded su	bset of M			
	C) f(A) is closed su	bset of M	B) A is closed subs	et of M			

87. If A is not bounded subset of a metric space M then diam (A) is -----. A) less than zero **B**) 0 **C**) 1 D) infinity 88. The metric space [a, b] with absolute value metric is -----. A) complete and not totally bounded B) totally bounded and not complete C) both complete and totally bounded D) neither complete nor totally bounded 89. If M is connected metric space then-----. A) M has a proper subset which is both open and closed B) M has a no proper subset which is both open and closed C) M is not open D) M is not closed A subset $C = \{0, 1, 2, \dots, 100\}$ in discrete metric space is-----. 90. C) need not be open A) connected B) open D) compact 91. Consider the following statements: I) Set of integers forms an abelian group w.r.t. the usual addition. II) Set of integers forms a group w.r.t. the usual multiplication. Then A) Only I) is true

- B) Only II) is true
- C) Both I) and II) are true
- D) Both I) and II) are false

92. $o(S_3) = _$ ____.

- A) 1
- B) 3
- **C**) 0
- D) 6

93. Let G be a group and let $a \in G$. Consider the following statements:

I) $cl(a) = \{a\}$. II) $a \in Z(G)$. Then A) Only I) \Rightarrow II) B) Only II) \Rightarrow I) C) Neither I) \Rightarrow II) nor II) \Rightarrow I) D) I) \Leftrightarrow II)

94. Consider the following statements:

I) R is an integral domain.
II) R is a field.
Then
A) Only I) ⇒ II)
B) Only II) ⇒ I)
C) Neither I) ⇒ II) nor II) ⇒ I)
D) I) ⇔ II)

95. Characteristic of a ring of integers is _____. A)1 B)2 C) 0 D)a prime number.

96. Which of the following statement is **not** correct for the ring

 $Z_4 = \{0, 1, 2, 3\}$ under addition and multiplication modulo 4?

A) 0 is idempotent B) 1 is idempotent C) 2 is idempotent D) 2 is nilpotent

97. An ideal $M \neq R$ of a ring R is said to be maximal ideal of R if whenever N is an ideal of R such that

then either N = M or N = R. A) $M \subseteq R \subseteq N$ B) $N \subseteq R \subseteq M$ C) $N \subseteq M \subseteq R$ D) $M \subseteq N \subseteq R$

98. Which of the following is alternating group A_3 ?

A) {(1 3), (1 2), (1 3 2)} B) {(1), (1 2 3), (1 3 2)} C) {(2), (1 2 3), (1 3)} D) {(1 2), (1 3), (2 3)}

- 99. Let G be a group and S be a subgroup of G. Let R be a ring and I be an ideal of R. Consider the following statements:
 - I) Quotient group $\frac{G}{s}$ can be defined.
 - II) Quotient ring $\frac{R}{I}$ can be defined. Then
 - A) Both I) and II) are true.
 - B) Only I) is true.
 - C) Both I) and II) are false.
 - D) Only II) is true.

100. Consider the field $Z_3 = \{0,1,2\}$ modulo 3.

If $f(x) = 1 + x^2 + x^3$, $g(x) = 2 + 2x^3 \in Z_3[x]$ over Z_3 , then f(x) + g(x) =_____. A) x^2 B) $1 + x^2 + x^3$ C) $1 + x^3$ D) $1 + x^2$

ROUGH WORK

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