Seat No.	

ENT - 60

Total No. of Pages : 18

M. Sc. Entrance Examination, 2024
MATHEMATICS
Sub. Code: 58716

Day and Date : Monday, 29-07-2024 **Total Marks : 100**

Time : 12.30 p.m. to 2.00 p.m.

Instructions :

1) All questions are compulsory.
2) Each question carries 1 mark.
3) Answers should be marked in the given OMR answer sheet by darkening the appropriate option.
4) Use black ball point pen only for marking the circle. Do not make any stray mark on the OMR Answer Sheet.
5) Follow the instructions given on OMR Sheet.
6) Rough work shall be done on the sheet provided at the end of question paper.
7) Only non programmable calculators are allowed.

1) If $f(x) = x^2$ on [2, 4] and $P = \left\{2, \frac{5}{2}, 3, \frac{7}{2}, 4\right\}$ is a partition of [2, 4], then L(f, P) =_____.

   A) $^{63}/_{3}$

   B) $^{63}/_{4}$

   C) $^{63}/_{8}$

   D) $^{126}/_{4}$

2) Let f be integrable function on $[a,\ b]$. For $x \in [a,\ b]$, let $F(x) = \int_1^x f(t)\ dt$. If f is continuous at x_0 in $(a,\ b)$, then $F'(x_0) =$ _________.

   A) $f''(x_0)$

   B) $f'(x_0)$

   C) $f(x_0)$

   D) $f(x_0) - f(a)$

3) If $P = \{a = t_0 < t_1 < t_2 < \cdots < t_n = b\}$ is a partition of $[a,\ b]$, then the mesh(P) = _____.

A) $\max\{t_k - t_{k-1} : k = 1,2,\dots,n\}$

B) $\min\{t_k - t_{k-1} : k = 1,2,\dots,n\}$

C) $\max\{t_k + t_{k-1} : k = 1,2,\dots,n\}$

D) $\min\{t_k + t_{k-1} : k = 1,2,\dots,n\}$

4) $\lim_{h\to 0} \frac{1}{h} \int_4^{4+h} e^{t^2}\, dt =$ ________.

A) $4e^{x^2}$

B) e^9

C) e^{16}

D) 4

5) The integral $\int_0^1 \frac{\sin\sqrt{x}}{\sqrt{x}} dx$ is ___________.

A) Improper integral of first kind

B) Improper integral of second kind

C) Improper integral of third kind

D) Not an improper integral

6) $\int_0^1 x^{m-1}(1-x)^{n-1}\, dx$ is convergent if ______.

A) $m > 0,\ n < 0$

B) $m < 0,\ n > 0$

C) $m > 0,\ n > 0$

D) $m < 0,\ n < 0$

7) $\int_0^1 \frac{\sec x}{x} dx$ is __________.

A) divergent

B) absolutely convergent

C) convergent

D) proper integral

8) The Cauchy Principal Value of $\int_{-\infty}^{\infty} e^{-x^2}\, dx$ is ______.

A) $\pi/2$

B) π

C) $\sqrt{\pi}/4$

D) $\sqrt{\pi}$

9) In the half range Fourier cosine series expansion of $f(x) = \pi - x$ in $[0,\ \pi]$, the value of a_0 is _______.

(A) 2π

(B) π

(C) $\dfrac{\pi^2}{2}$

(D) 0

10) In the Fourier series expansion of $f(x) = x\cos x$ in $[-\pi,\ \pi]$, the value of a_n is ____

(A) 2

(B) π

(C) $\dfrac{2\,(-1)^{n+1}}{n^2-1}$

(D) 0

11) Which of the following statement is **not** correct for any group G.

A) Identity element is unique.

B) Inverse of each $a \in G$ is unique.

C) $(a^{-1})^{-1} = a$ for all $a \in G$.

D) $(ab)^{-1} = a^{-1}b^{-1}$ for all $a, b \in G$.

12) $o(D_4) =$ ______.

A) 4

B) 8

C) 12

D) 4!

13) If $G = S_3$ then $G' =$ ______.

A) A_3

B) S_3

C) $\{e\}$

D) D_4

14) Let G be a finite group and let $a \in G$. Consider the following statements:

I) $o(cl(a)) = 1$.

II) $a \in Z(G)$.

Then

A) Only I) $\Rightarrow$ II)

B) Only II) $\Rightarrow$ I)

C) Neither I) $\Rightarrow$ II) nor II) $\Rightarrow$ I)

D) I) $\Leftrightarrow$ II)

15) A ring R is called ______ if $x^2 = x$ for all $x \in R$.

A) an integral domain

B) division ring

C) a Boolean ring

D) field

16) An element a in a ring R is called _____ if $a^n = 0$ for some integer n.

A) generator

B) zero divisor

C) idempotent

D) nilpotent

17) Let Z be the ring of integers under usual addition and multiplication. Consider the following statements:

I) Set of even integers is an ideal of Z

II) Z is a field.

Then

A) Only I) is true

B) Only II) is true

C) Both I) and II) are true

D) Both I) and II) are false

18) Let R and R' be two rings. Let 0 and $0'$ be zeros of R and R' respectively. If $f: R \to R'$ be a homomorphism, then $Ker\ f =$ _____.

A) $\{a \in R' | f(a) = a\}$

B) $\{a \in R | f(a) = 0'\}$

C) $\{a \in R | f(a) = a\}$

D) $\{a \in R' | f(a) = 0\}$

19) Let R be a ring and I be an ideal of R. If $I = R$, then quotient ring $\frac{R}{I} \cong$_____.

A) R B) $\{0\}$ C) I D) Z_p

20) Consider the following statements for a group S_3:

I) S_3 is not abelian.

II) $o(S_3) = 3!$. Then

A) Both I) and II) are true. B) Both I) and II) are false.

C) Only II) is true. D) Only I) is true.

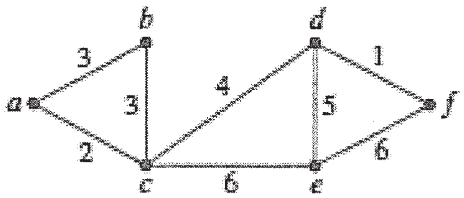
21) Let $G = (V, E)$ be a graph, where $V = \{v_1, v_2, v_3, v_4, v_5\}$ and $E = \{(v_1, v_4), (v_2, v_3), (v_3, v_1), (v_4, v_2), (v_5, v_2), (v_4, v_1), (v_4, v_4)\}$ then the degree of the vertex v_4 is

A) 3 B) 4 C) 5 D) 2

22) A graph G has 26 edges and degree of each vertex is k, then which of the following is possible number of vertices of G?

A) 25 B) 10 C) 13 D) 20

23) The number of distinct minimum spanning trees for the following graph are

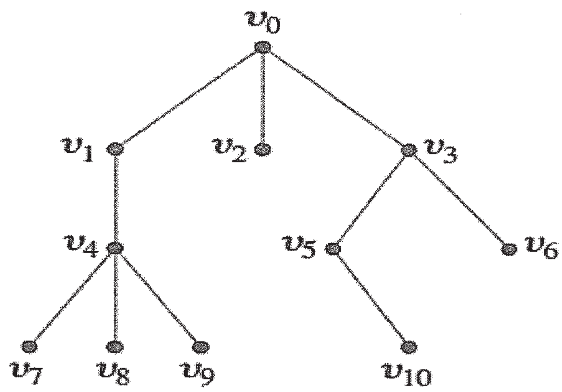


A) 2 B) 3 C) 6 D) 5

24) In an Euler graph the degree of every vertex is

A) same B) even C) odd D) prime number

25) Consider the tree with root v_0 shown below



The height of the rooted tree is

A) 5 B) 3 C) 4 D) 9

26) Which of the following is not the rule of inference?
A) Modus Ponens B) Transitivity C) Elimination D) Contradiction

27) Let P: We should be honest., Q: We should be dedicated., R: We should be overconfident. Then 'We should be honest or dedicated but not overconfident.' is best represented by?

A) $\sim P \vee \sim Q \vee R$ B) $P \wedge \sim Q \wedge R$ C) $P \vee Q \wedge R$ D) $P \vee Q \wedge \sim R$

28) Binary equivalent of the octal number 173 is

A) 1110111 B) 1111011

C) 1101111 D) 1101011

29) A-F system is used in which of the following number systems?
A) Binary B) Octal C) Hexa-Decimal D) Decimal

30) Addition of the binary numbers 101101 and 11101 is
A) 1001010 B) 1111010 C)1001110 D) 1110010

31) Which of the following is not the characteristics of the canonical form of L. P. P......

A) objective function is of maximization type

B) all constraints are of ($\leq$) type

C) all variables x_i are non-negative

D) all constraints are expressed as equations

32) The IBFS to the following transportation problem

Distribution Centres

		D_1	D_2	D_3	D_4	Supply
	P_1	19	30	50	12	7
Plants	P_2	70	30	40	60	10
	P_3	40	10	60	20	18
Demand		5	8	7	15	

by using North-West Corner method is ....
A) 975 B)875 C)985 D)675

33) The transportation problem deals with the transportation of ______________.
A) a single product from a source to several destinations
B) a single product from several sources to several destinations
C) a single product from several sources to a destination
D) a multi -product from several sources to several destinations

34) Which of the following is a method for improving an initial solution in a transportation problem ....

A) northwest-corner
B) intuitive lowest-cost
C) southeast-corner rule
D) stepping-stone

35) The purpose of a dummy source or dummy destination in a transportation problem is to

A) prevent the solution from becoming degenerate.
B) obtain a balance between total supply and total demand.
C) make certain that the total cost does not exceed some specified figure.
D) provide a means of representing a dummy problem.

36) In L. P. P. when does feasibility change ....

A) addition of variable
B) change in objective functions coefficient
C) change in right hand side of feasible region
D) feasibility does not change

37) The cost of a slack variable is ______________.

A) 0 B) 1 C) 2 D) -1

38) As for maximization in assignments problem, the objective is to maximize the ....

A) profit B) optimization C) cost D) None of the above

39) For the following game

Player B

		B_1	B_2
Player A	A_1	20	-6
	A_2	-4	3

The value of game is .....

A) 10/11 B) 13/11 C) 12/11 D) 14/11

40) There are five jobs, each of which must go through the two machines A and B in the order AB. Processing times (hours) are given below:

Job	1	2	3	4	5
Time for A	10	2	18	6	20
Time for B	4	12	14	16	8

the total elapsed time T is ....

A) 30 B) 40 C) 50 D) 60

41) In any metric space $<M, \rho>$ the Cauchy sequence ----------------.

A) is always convergent sequence B) need not be convergent sequence
C) is not convergent sequence D) none of these

42) If M is the closed interval $[-2, 2]$ with absolute value metric, then open ball B $[0 ; 3]$ is the interval ---------------.

A) $[-3,3]$ B) $(0, 3)$ C) $(-2, 2)$ D) $[-2,2]$

43) Which of the following is closed subset of an absolute metric space?

A) {1} B) $(1, \infty)$ C) $(1,2]$ D) none of

44) In any metric space inverse image of closed set is ----------------.

A) need not be closed set B) is closed set

C) is open set D) neither open nor closed

45) Let $< M, \rho >$ be a metric space. The subset A of M is totally bounded if and onl

A) there exists a sequence of points of A that contains a Cauchy subsequence

B) every sequence of points of A contains a Cauchy subsequence

C) there exists a sequence of points of A that contains a convergent subsequen

D) every sequence of points of A contains a convergent subsequence

46) For a subset A = (3, 5) of R,

(I) diam(A) = 1 in R_d.

(II) diam(A) = 2 in R^1.

Then ----------------.

A) only statement (I) is true

B) only statement (II) is true

C) both statements (I) and (II) are false

D) both statements (I) and (II) are true

47) If f is a continuous function of the compact metric space M_1 into a metric space M_2, then the range $f(M_1)$ of f is ----------------.

A) open subset of M_2 B) bounded subset of M_2

C) connected subset of M_2 D) closed subset of M_2

48) If A is not a connected subset of R^1 then ----------------.

A) A may be a singleton set

B) A may be union of intervals with nonempty intersection

C) A may be an interval

D) A may be union of intervals with empty intersection

49) A set C = { 1, 2, . . . , 1000} in a discrete metric space is---------------.

A) compact B) open C) need not be open D) connected

50) Let $< M, \rho >$ be a metric space. T: M → M is a contraction on M then which of the following is true?

A) ρ (Tx, Ty) ≤ 2 ρ(x, y) B) ρ (Tx, Ty) ≤ $\frac{3}{2}$ ρ(x, y)

C) ρ (Tx, Ty) ≤ $\frac{2}{3}$ ρ(x, y) D) ρ (Tx, Ty) ≥ 3 ρ(x, y)

51) $L\left\{\frac{1}{\sqrt{t}}\right\} =$ ________.

A) $\sqrt{\pi s}$ B) $\sqrt{\frac{s}{\pi}}$ C) $\sqrt{\frac{\pi}{s}}$ D) $\sqrt{\frac{1}{\pi s}}$

52) $L\{\sin^2 2t\} =$ ________.

A) $\frac{2}{s^2+4}$ B) $\left(\frac{2}{s^2+4}\right)^2$ C) $\frac{s}{s^2+4}$ D) $\frac{8}{s(s^2+16)}$

53) $L\left\{\int_0^t \cos 4u \, du\right\} =$ ________.

A) $\frac{1}{s^2+16}$ B) $\frac{s}{s^2+16}$ C) $\frac{1}{s(s^2+16)}$ D) $\frac{4}{s^2+16}$

54) If $F(t+2)$ is a Heaviside unit step function then $L\{F(t+2)\} =$ ________.

A) e^{-2s} B) $\frac{e^{2s}}{s}$ C) $\frac{e^{-2s}}{s}$ D) e^{2s}

55) $L^{-1}\left\{\frac{1}{s+1} + \frac{6}{s^4}\right\} =$ ________.

A) $e^{-t} + t^3$ B) $e^{-t} + \frac{t^3}{6}$ C) $e^{-t} + \frac{t^4}{4}$ D) $e^{-t} + t^3$

56) If $L^{-1}\{f(s)\} = 4\sin t$ then $L^{-1}\{f(4s)\} =$ _____

A) $\sin t$ B) $\cos t$ C) $\frac{1}{4}\sin t$ D) $\frac{1}{4}\sin\left(\frac{t}{4}\right)$

57) If $L^{-1}\{f(s)\} = e^{-t}$ then $L^{-1}\left\{\frac{d}{ds} f(s)\right\} =$ _____

A) te^{-t} B) $t\,e^{t}$ C) $-e^{-t}$ D) $-te^{-t}$

58) If $f_c(s)$ is Fourier cosine transform of $F(X)$ then the Fourier cosine transform of $F\left(\frac{X}{a}\right)$ is ________

A) $af_c(as)$ B) $\frac{1}{a}f_c(as)$ C) $af_c\left(\frac{a}{s}\right)$ D) $\frac{1}{a}f_c\left(\frac{a}{s}\right)$

59) In order to find infinite Fourier transform of $f(x)$, the function $f(x)$ must be ....

A) continuous in $(-\infty, \infty)$ B) continuous in $(0, \infty)$

C) integrable in $(-\infty, \infty)$ D) continuous and integrable in $(-\infty, \infty)$

60) If $F(x)$ is continuous and $F'(x)$ is piecewise continuous, then $F_s\{F'(x)\} =$ ________

A) $F_c\{F(x)\}$ B) $-F_c\{F(x)\}$

C) $-s^2 F_c\{F(x)\}$ D) $-s\, F_c\{F(x)\}$

61) If W is a subspace of V then $L(W) =$

A) W B) V C) $\{0\}$ D) ϕ

62) If $u = (1,0,0,0)$ then norm of u with respect to Euclidean inner product in $\mathbb{R}^4$ is

A) 1 B) 2 C) 3 D) 4

63) If dim $V = n$ and $S = \{v_1, v_2, \dots, v_n\}$ spans V then S is ... ... of V.

A) a subspace B) a basis

C) a linearly dependent subset D) the smallest subspace

64) If $T(0,1) = (0,3)$ then ________ is an eigen value of T.

A) 0 B) 1 C) 2 D) 3

65) Let $\{u, v, w\}$ be a linearly independent set in a vector space. Then which of the following is correct?

A) u is a linear combination of v and w.
B) $\{u, v, u+v\}$ is linearly independent.
C) $au + bv + cw = 0$ for some nonzero scalars a, b and c.
D) $\{u, u+v, u+v+w\}$ is linearly independent.

66) Let V be the inner product space of real polynomials of degree at most 2 with respect to the inner product defined by

$$\langle f, g \rangle = \int_0^1 f(x) \cdot g(x)dx$$

If $f(x) = x$ and $g(x) = x^3$, then $\langle f, g \rangle =$

A) 1 B) -1 C) 5 D) $\frac{1}{5}$

67) Let $S = \{(1,0,2), (1,1,2)\}$. The value of k for which the vector $(2k+2, 1, 5)$ belongs to the linear span of S is

A) 1 B) – 1 C) 4 D) 1/4

68) The characteristic polynomial of the matrix $\begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}$ is

A) $x^2 - 3x + 1$ B) $x^2 + 3x - 10$
C) $x^2 - 3x$ D) $x^2 - 3x - 10$

69) If $T: V \to W$ and $S: W \to U$ are two linear transformations such that ST is onto then

A) S is onto
B) T is one - one
C) S is one - one
D) T is onto

70) If V is an Inner product space and $x, y \in V$ then $\| x + y \|^2 + \| x - y \|^2 =$

A) $2(\| x \|^2 - \| y \|^2)$
B) $\| x \|^2 + \| y \|^2$
C) $2(\| x \|^2 + \| y \|^2)$
D) $\| x \|^2 - \| y \|^2$

71) $1 + xa + \frac{x^2}{2!}a^2 + \frac{x^3}{3!}a^3 + \frac{x^4}{4!}a^4 + \cdots$ is power series expansion of __________.

A) $\sin ax$
B) e^{ax}
C) $cosax$
D) e^x

72) $\lim_{x \to 0^-} \left(\frac{1}{1 - e^{\frac{-1}{x}}} \right) =$__________.

A) 0
B) 1
C) -1
D) 2

73) If f(x)=$\frac{x - |x|}{x}$ at x≠0 and f(x)=2 at x=0 then at x=0 function has__________.

A) Removable Discontinuity
B) Discontinuity of first kind
C) Discontinuity of second kind
D) Infinite Discontinuity

74) Which of the following is not exact differential equation?

A) $(x^2 + y^2 + 1)dx + 2xydy = 0$
B) $(y^2 - ax)dy + (x^2 - ay)dx = 0$
C) $2xydx + (y^2 - x^2)dy = 0$
D) $3x^2ydx + x^3dy = 0$

75) The P.I. of $(D - a)^2 y = e^{ax}$ is ....

A) $\frac{e^{ax}}{2a}$
B) $\frac{x^2}{2}e^{ax}$
C) $\frac{x}{2}e^{ax}$
D) $\frac{x^3}{3!}e^{ax}$

76) The solution of the homogeneous linear equation $x^2\frac{d^2y}{dx^2} + 4x\frac{dy}{dx} + 2y = 0$ is ....

A) $y = \frac{c_1}{x^4} + \frac{c_2}{x}$
B) $y = \frac{c_1}{x} + \frac{c_2}{x}$
C) $y = \frac{c_1}{x} + \frac{c_2}{x^2}$
D) $y = \frac{c_1}{x^2} + \frac{c_2}{x^2}$

77) If $z = \left(\frac{\sqrt{x} - \sqrt{y}}{\sqrt{x} + \sqrt{y}} \right)$ then $x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} =$_______________.

A) $\sqrt{z}$
B) z
C) 0
D) -2

78) General solution of the differential equation $\frac{dx}{x} = \frac{dy}{y} = \frac{dz}{z}$ is ....

A) $\Phi(xy, yz) = 0$
B) $x = c_1 y,\ y = c_2 z$
C) $\Phi\left(\frac{x}{y}, yz\right) = 0$
D) $\Phi(xyz, yz) = 0$

79) Method of taking one variable as constant is useful in solving........... equation ·

A) Simultaneous
B) Total differential
C) homogeneous linear
D) linear equation with constant coefficients

80) By inspection method the solution of $ayzdx + bzxdy + cxydz = 0$ is .... ·

A) $x^a y^b z^c = k$
B) $xyz = abc$
C) $x^a + y^b + z^a = k$
D) $\frac{x^2y^2z^2}{abc} = 1$

81) The amplitude of $\frac{1+i\sqrt{3}}{\sqrt{3}+i}$ is.......

A) $\pi/6$
B) $\pi/4$
C) $\pi/3$
D) None of these

82) If $z = 1 + i\sqrt{3}$ then $|\arg z| + |\arg \bar{z}| = \cdots \ldots \ldots$

A) $\pi/3$
B) $2\pi/3$
C) 0
D) $\pi/2$

83) If $f(z) = \bar{z}$ is ....

A) Continuous for every z not differential for any z
B) Continuous for some values of z differentiable for every z
C) discontinuous for every z , differential for every z
D) neither continuous nor discontinuous

84) If C is given by the equation $|z - a| = R$ then the value of $\int_C \frac{dz}{z-a}$ is .....

A) πi
B) $2\pi i$
C) $\frac{\pi i}{2}$
D) None of these

85) The residue of $z^2 \sin\frac{1}{z}$ at z= 0 is ....

A) $\frac{1}{12}$
B) $\frac{1}{6}$
C) 6
D) $-\frac{1}{6}$

86) The residue of $z^2 e^{1/z}$ at z = 0 is .....

A) 1/12 B) 1/6 C)6 D)12

87) If $I = \int \frac{dz}{z-2}$ then value of I along the curve $|z-2| = 3$ *is* ... .

A) 0
B) 2 π
C) 2πi
D) None of these

88) If $u(x,y) = x^2 - y^2$ then corresponding analytic function $f(z) =$________ .

A) $z^2 + C$ B) $z^3 + C$ C) $z + C$ D) $z^3 + iC$

89) The singularity of function $\frac{1}{\sin z - \cos z}$ at $z = \frac{\pi}{4}$ is

A) Simple pole
B) Double pole
C) Singularity
D) None of these

90) Residue of $\frac{z^2-2z}{(z+1)^2(z^2+4)}$ at double pole $z = -1$ is

A) $\frac{4}{5}$
B) $-\frac{4}{5}$
C) $-\frac{14}{25}$
D) $\frac{14}{25}$

91) Let A ={1, 2, 3 } and R ={(1, 1), (2, 3), (3, 1)} be a relation on A then R is …

A) reflexive B)symmetric

C) anti-symmetric D) transitive

92) Acyclic group of order 7 has … no. of generators.

A) 7 B) 6 C) 5 D) 4

93) If H is a subgroup of G and O(H) = 6, O(G) = 54 then [G : H] = …

A) 6 B) 9 C) 3 D) 2

94) If G is a group and $\in G$, then a subset $\{x \in G/xa = ax\}$ is called…

A) Center of G B) normalizer of 'a' in G

C) right coset of 'a' in G D) none of these

95) Every finite group is isomorphic to ….

A) abelian group B) a cyclic group

C) a permutation group D) none of these

96) Which of the following are zero divisors in a ring $\langle Z_{12}, \oplus_{12}, \otimes_{12} \rangle$

A) 6, 2 B) 6, 5 C) 4, 7 D) 5 ,7

97) The greatest lower bound of the set $\left\{x \in Q / x = \frac{(-1)^n}{n}, n \in N\right\}$ is…

A)0 B) 1 C) -1 D) 2

98) If $\{S_n\} = \{1, -1, 1, -2, 1, -3, \dots\}$, then $\lim_{n \to \infty} sup\, S_n = \cdots$

A) 0 B) 1 C) -1 D) 2

99) The positive p- series $\sum \frac{1}{n^2}$ is …

A)convergent B) divergent

C)neither convergent nor divergent D) none of these

99) The positive p- series $\sum \frac{1}{n^2}$ is …

A)convergent B) divergent

C)neither convergent nor divergent D) none of these

100) The series $\sum \left(\frac{1}{2}\right)^n$ is ….

A)convergent B) divergent

C)neither convergent nor divergent D) none of these

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