SHIVAJI UNIVERSITY, KOLHAPUR

M.Sc. (Mathematics) Entrance Examination (2024-25) Syllabus

Syllabus for Entrance Examination of M.Sc. (Mathematics) for the academic year 2024-25, will be based on theory courses of B.Sc. (Mathematics).

The distribution of weightage of marks will be as follows:

Sr. No.	Topic	weightage
1	Mathematics courses of T.Y.B.Sc. (Mathematics)	80 %
2	Mathematics courses of S.Y.B.Sc. (Mathematics)	10 %
3	Mathematics courses of F.Y.B.Sc. (Mathematics)	10 %

Note that:

1. Duration of Examination : 3 hours
1. Total number of multiple choice questions (MCQs) : 100
2. Maximum Marks: 100
3. All MCQs are compulsory.

Syllabus For
B.Sc. Part -III (Mathematics)
SEMESTER V AND VI

Title of Course : Mathematical Analysis

Unit -1 : Riemann Integration

Definition of Riemann integration, Inequalities for lower and upper Darboux sums, Necessary and sufficient conditions for Riemann integrability, Definition of Riemann integration by Riemann sum and equivalence of the two definitions, Riemann integrability of monotonic functions and continuous functions, Algebra and properties of Riemann integrable functions, First and second fundamental theorems of integral calculus, and the integration by parts.

Unit -2 : Improper Integrals and Fourier Series

Improper Integrals: Definition of improper integral of first kind, Comparison test, – test for Convergence, Absolute and conditional convergence, Integral test for convergence of series, Definition of improper integral of second kind and some tests for their convergence, Cauchy principle value.

Fourier Series: Definition of Fourier series and examples on the expansion of functions in Fourier series, Fourier series corresponding to even and odd functions, half range Fourier series, half range sine and cosine series

Recommended Books:

1. **Kenneth.A.Ross**, Elementary Analysis: The Theory of Calculus, Second Edition, Undergraduate Textsin Mathematics, Springer, 2013.
   (Chapter 6, Art. 32.1 to 32.11, 33.1 to 33.6 and 34.1 to 34.4)
2. **D Somasundaram and B Choudhary**, First Course in Mathematical Analysis, Narosa Publishing House New Delhi, Eighth Reprint 2013 (Chapter 8, Art. 8.5 and Chapter 10, Art. 10.1)

Reference Books:

1. **R.R.Goldberg**, Methods of Real Analysis, Oxford & IBH Publishing Co. Pvt. Ltd., New Delhi.
2. **R.G.Bartle and D.R.Sherbert**, Introduction to Real Analysis, Wiley India Pvt. Ltd., Fourth Edition 2016.
3. **Shanti Narayan and Dr.M.D.Raisinghania,** Elements of Real Analysis, S.Chand& Company Ltd. New Delhi, Fifteenth Revised Edition 2014
4. **Shanti Narayan and P.K.Mittal,** A Course of Mathematical Analysis, S.Chand& Company Ltd. New Delhi, Reprint 2016.
5. **Kishan Hari**, Real Analysis, PragatiPrakashan, Meerut, Fourth Edition 2012.

Title of Course : Abstract Algebra

Unit -1: Groups and Rings

Groups: Definition and examples of groups, group S_3 and Dihedral group D_4, Commutator subgroups andits properties, Conjugacy in group and class equation.
Rings: Definition and example of Rings, Ring with unity. Zero divisor, Integral Domain, Division Ring, Field, Boolean ring, Subring, Characteristic of a ring: Nilpotent and Idempotent elements. Ideals, Sum of two ideals, Examples. Simple Ring**.**

Unit-2: Homomorphism and Imbedding of Ring , Polynomial Ring and Unique Factorization Domain.

Quotient Rings, Homomorphism, Kernel of Homomorphism, Isomorphism theorems, imbedding of Ring, Maximal Ideals, Polynomial Rings, degree of Polynomial, addition and multiplication of Polynomials and their properties, UFD, Gauss' Lemma.

Recommended Books:

1. **Vijay K. Khanna, S.K. Bhambri**, A Course In Abstract Algebra, Vikas publishing House Pvt.Ltd., New –Delhi-110014, Fifth Edition 2016.
   (Chap. 3 Art. The Dihedral Group, commutator, Chap. 4 Art. Conjugate elements, Chap.7 Art. Subrings, characteristic of a ring, Ideals, Sum of Ideals, Chap. 8 Art. Quotient rings, Homomorphisms, Embedding of Rings, More on Ideals, Maximal Ideals, Chap 9 Polynomial Rings, Unique Factorization Domain.)

Reference Books:

1. **Jonh B. Fraleigh**, A First Course in Abstract Algebra Pearson Education, Seventh Edition(2014).
2. **Herstein I. N**, Topics in Algebra, Vikas publishing House,1979.
3. **Malik D. S. Moderson J. N. and Sen M. K**., Fundamentals of Abstract Algebra, McGrew Hill,1997.
4. **Surjeet Sing and Quazi Zameeruddin**, Modern Algebra, Vikas Publishing House,1991.
5. **N. Jacobson**, Basic Algebra Vol. I&II, Freeman and Company, New York 1980.

Title of Course: Optimization Techniques

Unit-1 Network optimization models :

Introduction, Formulation of Linear Programming Problems., Graphical methods for Linear Programming problems. General formulation of Linear Programming problems, Slack and surplus variables, Canonical form, Standard form of Linear Programming problems. Transportation problem: Introduction, Mathematical formulation ,Matrix form of Transportation problem. Feasible solution, Basic feasible solution and optimal solution, Balanced and unbalanced transportation problems.Methods of Initial basic feasible solutions: North west corner rule [Stepping stone method], Lowest cost entry method [Matrix minima method], Vogel's Approximation method [Unit Cost Penalty method] ,The optimality test.[MODI method], Assignment Models :Introduction ,Mathematical formulation of assignment problem, Hungarian method for assignment problem. Unbalanced assignment problem. Travelling salesman problem.

Unit-2 Quantitative techniques:

Game theory: Basic definitions , Minimax [Maximin] Criterion and optimal strategy, Saddle point , optimal strategy and value of game. Solution of games with saddle point. Fundamental theorem of game theory [Minimax theorem] , Two by two (2 X 2) games without saddle point. Algebraic method of Two by two (2 X 2) games. Arithmetic method of Two by two (2 X 2)games. Graphical method for 2 x n games and m x 2 games. Principle of dominance, Job sequencing : Introduction. Terminology and notations. Principal assumptions. Solution of sequencing problems. Processing n jobs through 2 machines. Processing n jobs through 3 machines. Processing 2 jobsthrough m machines. Processing n jobs through m machines.

Recommended Books:

1. Sharma S.D., Operations Research - Theory Methods and Applications"Kedarnath, Ramnath Meerut,Delhi Reprint 2015.

Reference Books:

1. Mohan, C. and Deep, Kusum, Optimization Techniques, New Age,2009.
2. Mittal, K. V. and Mohan, C., Optimization Methods in Operations, Research and Systems Analysis,New Age, 2003.
3. Taha, H.A. : Operations Research – An Introduction, Prentice Hall, (7th Edition), 2002.
4. Ravindran, A. , Phillips, D. T and Solberg, J. J., Operations Research: Principles and Practice,John Willey and Sons, 2nd Edition, 2009.
5. J.K.Sharma : Operation Research: Theory and Applications, Laxmi Publications, 2017.
6. KantiSwarup,P.K.Gupta and Manmohan,Operation Research, S.Chand& Co.
7. G.Hadley: Linear programming , Oxford and IBH Publishing Co.

Title of Course: Integral Transforms

Unit: 1 Laplace and Inverse Laplace Transform.

Laplace Transform **:** Definitions; Piecewise continuity, Function of exponential order, Function of class A ,Existence theorem of Laplace transform. Laplace transform of standard functions. First shifting theorem and Second shifting theorem and examples, Change of scale property and examples, Laplace transform of derivatives and examples, Laplace transform of integrals and examples. Multiplication by power of t and examples. Division by t and examples. Laplace transform of periodic functions and examples. Laplace transform of Heaviside's unit step function.Inverse Laplace Transform: Definition Standard results of inverse Laplace transform, Examples, First shifting theorem and Second shifting theorem and examples. Change of scale property and Inverse Laplace of derivatives, examples. The Convolution theorem and Multiplication by S, examples. Division by S, inverse Laplace by partial fractions, examples, Solving linear differential equations with constant coefficients by Laplace transform.

Unit 2 Fourier Transform

The infinite Fourier transform and inverse: Definition examples Infinite Fourier sine and cosine transform and examples. Definition: Infinite inverse Fourier sine and cosine transform and examples. Relationship between Fourier transform and Laplace transform. Change of Scale Property and examples. Modulation theorem. The Derivative theorem. Extension theorem. Convolution theorem and examples. Finite Fourier Transform and Inverse, Fourier Integrals : Finite Fourier sine and cosine transform with examples. Finite inverse Fourier sine and cosine transform with examples. Fourier integral theorem. Fourier sine and cosine integral (without proof) and examples.

Recommended Books

1. J. K. Goyal, K.P.Gupta, Laplace and Fourier Transforms, A Pragati Edition (2016).

Reference Books:

1. Dr. S. Shrenadh, Integral Transform, S.Chand Prakashan.
2. B. Davies, Integral Transforms and Their Applications, Springer Science Business Media LLC (2002).
3. Murray R. Spiegel, Laplace Transforms, Schaum's outlines.

Title of Course: Metric spaces

Unit: 1 Limits and Continuous Functions on Metric Spaces

Limit of a function on the real line, Metric Spaces, Limits in Metric Spaces, Functions continuous at a point on the real line, Reformulation, Functions continuous on a metric space, Open Sets, Closed Sets, More about open sets.

Unit 2 Connectedness, Completeness and Compactness

Connected Sets, Bounded sets and totally bounded sets, Complete metric spaces,Compact metric spaces, Continuous functions on compact metric space.

Recommended Books

1. R. R. Goldberg, Methods of Real Analysis,Oxford and IBH Publishing House (2017).

Reference Books:

1. T. M. Apostol, Mathematical Analysis, Narosa Publishing House (2002) .
2. Satish Shirali, H. L. Vasudeva, Mathematical Analysis, Narosa Publishing House (2013).
3. D. Somasundaram, B. Choudhary, First Course in Mathematical Analysis, Narosa Publishing House, (2018).
4. W. Rudin, Principles of Mathematical Analysis, McGraw Hill Book Company (1976).
5. Shantinarayan, Mittal, A Course of Mathematical Analysis, S. Chand and Company (2013).
6. J.N. Sharma, Mathematical Analysis-I, Krishna Prakashan Mandir, Meerut (2014).
7. S.C. Malik, Savita Arora, Mathematical Analysis, New age International Ltd(2005).
8. B. Davies, Integral Transforms and Their Applications, Springer Science Business Media LLC (2002).

Title of Course: Linear Algebra

Unit: 1 Vector Spaces and Linear Transformations

Vector space: Subspace**,** Sum of subspaces, direct sum, Quotient space, Homomorphism or Linear transformation, Kernel and Range of homomorphism, Fundamental Theorem of homomorphism, Isomorphism theorems, Linear Span, Finite dimensional vector space, Linear dependence and independence, basis, dimension of vector space and subspaces. Linear Transformation: Rank and nullity of a linear transformation, Sylvester's Law, Algebra of LinearTransformations , Sum and scalar multiple of Linear Transformations. The vector space of Homomorphisms, Product (composition) of Linear Transformations, Linear operator, Linear functional,Invertible and non-singular Linear Transformation, Matrix of Linear Transformations and its examples.

Unit 2 Connectedness, Completeness and Compactness

Inner product spaces: Norm of a vector, Cauchy- Schwarz inequality, Orthogonality, Generalized Pythagoras Theorem, orthonormal set, Gram-Schmidt orthogonalization process, Bessel's inequality, Eigen values and Eigen vectors: Eigen space, Characteristic Polynomial of a matrix and remarks on it,similar matrices, Characteristic Polynomial of a Linear operator, Examples and real life (Predatory – Prey problem), examples on eigen values and eigen vectors.

Recommended Books

1. Khanna V. K. and Bhambri S. K., ACourse in Abstract Algebra, Vikas Publishing House PVT Ltd., New Delhi , 2016, 5th edition.

Reference Books:

1. H. Anton & C. Rorres, Elementary Linear Algebra (with Supplemental Applications), Wiley India Pvt. Ltd (Wiley Student Edition), New Delhi , 2016, 11th Edition.
2. S. Friedberg, A. Insel and L. Spence, Linear Algebra, Prentice Hall of India, 2014, 4th Edition.
3. Holfman K. and Kunze R., Linear Algebra, Prentice Hall of India, 1978.
4. Lipschutz S., Linear Algebra, Schaum's Outline Series, McGraw Hill, Singapore, 1981.
5. David Lay, Steven Lay, Judi McDonald, Linear Algebra and its Applications, Pearson Education Asia, Indian Reprint, 2016, 5th Edition.

Title of Course: Complex Analysis

Unit: 1 Analytic functions and Complex Integration

Basic algebraic and geometric properties of complex numbers, Function of complex variable, Limits, continuity and differentiation, Cauchy Riemann equations, Analytic functions and examples of analytic functions, Exponential function, Logarithmic function, Trigonometric function, Definite integrals of functions, Contours, Contour integrals and its examples, upper bounds for moduli of contour integrals, Cauchy-Goursat theorem and examples, Cauchy integral formula and examples, Liouville's theorem and the fundamental theorem of algebra.

Unit 2 Sequences, Series and Residue Calculus

Convergence of sequences and series of complex variables, Taylor series and its examples, Laurent series and its examples, absolute and uniform convergence of power series, Isolated singular points,Residues, Cauchy's residue theorem, Residue at infinity, The three types of isolated singularities, Residues at poles and examples, Zeros of analytic functions, Zeros and poles, Application of residuetheorem to evaluate real integrals.

Recommended Books

1. James Ward Brown and Ruel V. Churchill, Complex Variables and Applications, 8th Ed., McGraw – Hill Education (India) Edition, 2014. Eleventh reprint 2018.

Reference Books:

1. S. Ponnusamy, Foundations of Complex Analysis, Narosa Publishing House, Second Edition. 2005, Ninth reprint 2013.
2. Lars V Ahlfors, Complex Analysis, McGraw-Hill Education; 3 edition (January 1, 1979).
3. S.B.Joshi, T. Bulboaca and P. Goswamy, Complex Analysis, Theory and Applications, DeGruyter, Germany (2019).

Title of Course: Discrete Mathematics

Unit 1 : Mathematical Logic

The logic of compound statements: Statements, compound statements, truth values, logical equivalence, tautologies and contradictions, Conditional statements: Logical equivalences involving implication, negation. The contrapositive of a conditional statements, converse, inverse of a conditional statements, biconditional statements. Valid and invalid arguments: Modus Ponens and modus Tollens, Additional valid argument forms, rules of inferences, contradictions and valid arguments, Number system: Addition and subtraction of Binary, decimal, quintal, octal, hexadecimal number systems and their conversions.

Unit 2: Graphs and trees

Graphs : Definitions, basic properties, examples, special graphs, directed and undirected graphs, concept of degree, Trails, Paths and Circuits: connectedness, Euler circuits, Hamiltonian circuits, Matrix representation of graphs, Isomorphism of graphs, isomorphic invariants, graph isomorphism forsimple graphs.
Trees: Definitions and examples of trees, rooted trees, binary trees and their properties. spanning trees , minimal spanning trees, Kruskal' salgorithm , Prim's algorithm, Dijkstra's shortest path algorithm.

Recommended Book:

1. Susanna S. Epp, Discrete Mathematics with Applications,PWS Publishing Company, 1995. (Brooks/Cole, Cengage learning, 2011).

References Books :

1. Kenneth H. Rosen, Discrete Mathematics and its Applications, McGraw Hill, 2002.
2. J. P. Tremblay and R. Manohar, Discrete Mathematical Structure with Applications, McGraw–Hill.
3. V. Krishnamurthy, Combinatories : Theory and Applications", East-West Press.
4. Kolman, Busby Ross, Discrete Mathematical Structures, Prentice Hall International.
5. R M Somasundaram, Discrete Mathematical Structures, (PHI) EEE Edition 7.
6. A.B.P. Rao and R.V. Inamdar, A Graduate Text in Computer Mathematics, SUMS [1991]
7. Seymour Lipschutz and Marc Lipson, Discrete Mathematics, Schaum's Outlines Series, Tata McGraw -Hill.
8. Mathematical Foundations of Computer Science: professional publications, JNTU Hyderabad.
9. Liu C. L, Elements of Discrete Mathematics, McGraw – Hill.

Syllabus For
B.Sc. Part -II (Mathematics)
SEMESTER III AND IV

Title of Course: Real Analysis–I

Unit 1: Functions and Countable sets

1.1. Sets.

1.1.1. Revision of basic notions in sets.

1.1.2. Operations on sets:-Union, Intersection, Complement, Relative complement, Cartesian product of sets, Relation.

1.2. Functions

1.2.1. Definitions: Function, Domain, Co-domain, Range, Graph of a function, Direct image and Inverse image of a subset under a function. Examples of direct image and inverse image of a subset.

1.2.2. **Theorem:** If $f: A \to B$ and $X \subseteq B, Y \subseteq B$, then $f^{-1}(X \cup Y) = f^{-1}(X) \cup f^{-1}(Y)$

1.2.3. **Theorem:** If $f: A \to B$ and if $X \subseteq B$, $Y \subseteq B$, then $f^{-1}(X \cap Y) = f^{-1}(X) \cap f^{-1}(Y)$

1.2.4. **Theorem:** If $f: A \to B$ and $X \subseteq A$, $Y \subseteq A$, then $f(X \cup Y) = f(X) \cup f(Y)$

1.2.5. **Theorem:** If $f: A \to B$ and if $X \subseteq A, Y \subseteq A$, then $f(X \cap Y) \subseteq f(X) \cap f(Y)$

1.2.6. **Definitions:** Injective, Surjective and Bijective functions (1-1 correspondance) Inverse function.

1.2.7. **Proposition:** If $f: A \to B$ is injective and $E \subseteq A$, then $f^{-1}(f(E)) = E$.

1.2.8. **Proposition:** If $f: A \to B$ is surjective and $H \subseteq B$, then $f(f^{-1}(H)) = H$.

1.2.9. **Definition:** Composite function, Restriction and Extension of a function.

1.2.10. **Theorem:** Let $f: A \to B$ and $g: B \to C$ be functions and let H be a subset of C. Then $(g \circ f)^{-1}(H) = f^{-1}(g^{-1}(H))$.

1.2.11. **Theorem:** Composition of two bijective functions is a bijective function.

1.2.12. Examples

1.3. Mathematical Induction

1.3.1. **Principle of Mathematical Induction** (without proof), Well ordering property of natural numbers

1.3.2. **Principle of Mathematical Induction** (second version: Statement only), Principle of strong induction (Statement only).

1.3.3. Examples based on 1.3.1 and 1.3.2

1.4. Countable Sets

1.4.1. **Definitions:** Denumerable sets, Countable sets, uncountable sets.

1.4.2. **Examples of denumerable sets:** Set of Natural numbers, Set of

Integers, Set of even natural numbers and odd natural numbers.

1.4.3. **Proposition:** Union of two disjoint denumerable sets is denumerable.

1.4.4. **Theorem:** If A_m is a countable set for each $m \in \mathbb{N}$, then the union $A = \bigcup_{m=1}^{\infty} A_m$ is countable. (Countable union of countable sets is countable)

1.4.5. **Theorem:** The set of Rational numbers is denumerable.

1.4.6. **Theorem:** Any subset of countable set is countable.

1.4.7. **Theorem:** The closed interval $[0,1]$ is uncountable.

1.4.8. **Corollary:** The set of all real numbers is uncountable.

1.4.9. **Examples**

Unit 2 : The Real numbers

2.1. Algebraic and Order Properties of $\mathbb{R}$.

2.1.1. Algebraic properties of real numbers.

2.1.2. **Theorem:** Let $a, b, c \in \mathbb{R}$.

(a) If $a > b$ and $b > c$, then $a > c$

(b) If $a > b$, then $a + c > b + c$

(c) If $a > b$ and $c > 0$, then $ac > bc$. If $a > b$ and $c < 0$, then $ac < bc$

2.1.3. Theorem:

(a) If $a \in \mathbb{R}$ and $a \neq 0$, then $a^2 > 0$.

(b) $1 > 0$

(c) If $n \in \mathbb{N}$, then $n > 0$.

2.1.4. **Theorem:** If $a \in \mathbb{R}$ is such that $0 \leq a < \epsilon$ for every $\epsilon > 0$ then $a = 0$.

2.1.5. **Theorem:** If $ab > 0$, then either (i) $a > 0$ and $b > 0$ or (ii) $a < 0$ and $b < 0$

2.1.6. **Corollary:** If $ab < 0$, then either (i) $a < 0$ and $b > 0$ or (ii) $a > 0$ and $b < 0$

2.2. Inequalities

2.2.1. If $a \geq 0$, $b \geq 0$, then prove that

$a < b \Leftrightarrow a^2 < b^2 \Leftrightarrow \sqrt{a} < \sqrt{b}$.

2.2.2. **Arithmetic-Geometric mean inequality** (with proof).

2.2.3. **Bernoulli's inequality** (with proof).

2.3. Absolute Value and neighbourhood

2.3.1. **Definition:** Absolute value of a real number

2.3.2. Theorem:

(a) $|ab| = |a|.|b|$ for all $a, b \in \mathbb{R}$

(b) $|a|^2 = a^2$ for all $a \in \mathbb{R}$

(c) If $c \geq 0$, then $|a| \leq c$ if and only if $-c \leq a \leq c$

(d) $-|a| \leq a \leq |a|$ for all $a \in \mathbb{R}$

2.3.3. **Theorem (Triangle inequality):** If $a, b \in \mathbb{R}$, then $|a + b| \leq |a| + |b|$.

2.3.4. **Corollary:** If $a, b \in \mathbb{R}$, then (i) $||a| - |b|| \le |a - b|$ (ii) $|a - b| \le |a| + |b|$

2.3.5. **Corollary:** If $a_1, a_2, \dots, a_n$ are any real numbers then

$$|a_1 + a_2 + \dots + a_n| \le |a_1| + |a_2| + \dots + |a_n|$$

2.3.6. Examples on inequalities

2.3.7. **Definition:** ϵ - Neighbourhood.

2.3.8. **Theorem:** Let $a \in \mathbb{R}$. If x belongs to the neighbourhood $V_\epsilon(a)$ for every $\epsilon > 0$ then $x = a$.

2.4. Completeness property of $\mathbb{R}$

2.4.1. **Definitions:** Lower bound, Upper bound of a subset of $\mathbb{R}$, Bounded set, Supremum (least upper bound), Infimum (greatest lower bound).

2.4.2. The completeness property of $\mathbb{R}$ (The supremum property)

2.4.3. Applications of the supremum property.

2.4.4. **Theorem: (Archimedean Property)** If $x \in \mathbb{R}$, then there exists $n_x \in \mathbb{N}$ such that $x \le n_x$.

2.4.5. **Corollary:** If $S = \left\{\frac{1}{n} : n \in \mathbb{N}\right\}$, then $\inf S = 0$.

2.4.6. **Corollary:** If $t > 0$, then there exists $n_t \in \mathbb{N}$ such that $0 < \frac{1}{n_t} < t$.

2.4.7. **Corollary:** If $y > 0$, then there exists $n_y \in \mathbb{N}$ such that $n_y - 1 < y < n_y$.

2.4.8. **Theorem:** There exists a positive real number x such that $x^2 = 2$.

2.4.9. **Theorem: (The Density theorem)** If x and y are any real numbers with $x < y$, then there exists a rational number $r \in \mathbb{Q}$ such that $x < r < y$.

2.4.10. **Corollary:** If x and y are real numbers with $x < y$, then there existsan irrational number z such that $x < z < y$.

2.5. Intervals

2.5.1. **Characterization theorem:** If S is a subset of $\mathbb{R}$ that contains at least two points and has the property

if $x, y \in S$ and $x < y$, thenthe closed interval $[x, y] \subseteq S$,

then S is an interval.

Recommended Book

1) **Introduction to Real Analysis**, Robert G. Bartle and Donald R. Sherbert, Wiley Student Edition, 2010.

Reference Books:

1) **Methods of Real Analysis**, R. R. Goldberg, Oxford and IBH Publishing House, New Delhi, 1970.

2) **A Basic Course in Real Analysis**, Ajit Kumar and S. Kumaresan, CRC Press, Taylor & Francis Group, 2014.

3) **Real Analysis**, HariKishan, Pragati Prakashan, fourth revised edition 2012.

4) **An Introduction to Real Analysis**, P. K. Jain and S. K. Kaushik, S. Chand& Co., New Delhi,2000.

Title of Course: Algebra–I

Unit1: Matrices and Relations

1.5. **Definitions:** Hermitian and Skew Hermitian matrices.
1.6. **Properties** of Hermitian and Skew Hermitian matrices.
1.7. Rank of a matrix, Row-echelon form and reduced row echelon form.
1.8. System of linear homogeneous equations and linear non-homogeneous equations.
  1.8.1. Condition for consistency
  1.8.2. Nature of the general solution
  1.8.3. Gaussian elimination and Gauss Jordon method (Using row-echelon form andreduced row echelon form).
  1.8.4. Examples on 1.4.1 and 1.4.3
1.9. The characteristic equation of a matrix, Eigen values, Eigen vectors of a matrix.
1.10. Cayley Hamilton theorem
1.11. Applications of Cayley Hamilton theorem (Examples).
1.12. **Relations**: Definition, Types of relations, Equivalence relation, Partial orderingrelation
1.13. **Examples** of equivalence relations and Partial ordering relations.
1.14. Digraphs of relations, matrix representation.
1.15. Composition of relations
1.16. Transitive closure, Warshall's algorithm
1.17. Equivalence classes, Partition of a set
  1.17.1. **Theorem:** Let $\sim$ be an equivalence relation on a set X. Then
    (a) For every $x \in X$, $x \in \overline{X}$
    (b) For every $x, y \in X$, $x \in \overline{Y}$ if and only if $\overline{X} = \overline{Y}$.
    (c) For every $x, y \in X$, either $\overline{x} = \overline{y}$ or $\overline{x} \cap \overline{y} = \emptyset$.
  1.17.2. Equivalence class Theorem

Unit2 : Groups

2.1. **Definition** of Binary Operations and examples
2.2. Groups and its Properties
  2.2.1. **Definition** of Group, Semigroup, Abelian group, finite and infinite group, Quaternion group and Order of the group and examples
  2.2.2. **Theorem:** In a group G
    (i) Identity element is unique
    (ii) Inverses of each elements in G is unique
    (iii) $(a^{-1})^{-1} = a$ for all $a \in G$
    (iv) $(ab)^{-1} = b^{-1}a^{-1}$ for all $a, b \in G$.
  2.2.3. **Theorem:** If G is a group with binary operation $*$, then the left and right cancellation laws hold in G, that is $a * b = a * c$ implies b = c, and $b * a = c * a$ implies $b = c$ for $a, b, c \in G$.
  2.2.4. **Theorem:** If G is a group with binary operation *, and if a and b are any elements of G, then linear equations a * x = b and y * a = b have unique solutions in G.
2.3. Subgroups
  2.3.1. **Definition** of Subgroup, Improper and Proper subgroups,

Trivial subgroup and examples

2.3.2. **Theorem:** A subset H of a group G is a subgroup of G if and only if

(i) H is closed under the binary operation of G.

(ii) The identity e of G is in H,

(iii) For all $a \in H$ it is true that $a^{-1} \in H$ also.

2.3.3. **Theorem:** A non empty subset H of a group G is a subgroup of G if and only if for all $a, b \in H$, $a * b^{-1} \in H$.

2.3.4. **Theorem:** Intersection of any two subgroups of a group is again a subgroup.

2.3.5. **Definition** of Normalizer of an element in group G, Center of group G.

2.3.6. **Theorem:** If G is a group and $a \in G$, then the set $N(a) = \{ x \in G \mid xa = ax \}$ is a subgroup of G.

2.3.7. **Theorem:** If G is a group, then the set $C = \{ x \in G \mid xa = ax, \text{ for all } a \in G \}$ is the set of all the elements of G which commutes with every elements of G.

2.4. Cyclic Groups and its Properties

2.4.1. **Definition** of Cyclic group generated by an element, Cyclic subgroup of a group and examples

2.4.2. **Theorem:** If G is a group and $a \in G$ is a fixed element of G, then the set $H = \{ a^n \mid n \in Z \}$ is a subgroup of G.

2.4.3. **Definition** of Order of an element of a group and its properties.

2.4.4. **Theorem:** Every cyclic group is abelian.

2.4.5. **Theorem:** If a is a generator of a cyclic group G, so is a^{-1}.

2.4.6. **Theorem:** If a is a generator of a cyclic group G, then O(a) = O(G).

2.4.7. **Theorem:** If G is a finite group of order n containing an element of order n, then G is cyclic.

2.4.8. **Theorem:** If in a cyclic group <a> of order k, $a^m = a^n$ ($m \equiv n$), then $m \equiv n \pmod{k}$.

2.4.9. **Theorem:** Every subgroup of a cyclic group is cyclic.

2.4.10. **Theorem:** A cyclic group of order d has $\phi(d)$ generators.

2.5. Cosets

2.5.1. **Definition** of Left and Right Cosets in group G and examples

2.5.2. **Theorem:** If H is a subgroup of G, then

(i) Ha = H if and only if $a \in H$

(ii) Ha = Hb if and only if $ab^{-1} \in H$

(iii) Ha is a subgroup of G if and only if $a \in H$

2.5.3. **Theorem:** If H is a subgroup of G, then for all $a \in G$ $Ha = \{x \in G \mid x \equiv a \bmod H\}$.

2.5.4. **Theorem:** If H is a subgroup of G then there exists a one to one correspondence between any two right (left) cosets of H in G.

Recommended Books

1. Howard Anton—Elementary Linear Algebra, Fifth Edition John Wiley & Sons.
2. J. B. Fraleigh, A First Course in Abstract Algebra, Narosa Publishing House New Delhi.

Reference Books---

1. Kenneth Hoffman,Raykunze---Linear Algebra, Second Edition, PHI Learning Private LTD. New Delhi-110001-2010.
2. Vivek Sahai, Vikas Bist—Linear Algebra, Alpha Science International LTD. Pangboume.
3. I. N. Herstein-- Topics in Algebra, Wiley India Pvt. Ltd.
4. S. kumaresan—Linear Algebra,A Geometric Approach.

Title of Course: Real Analysis – II

UNIT 1: Sequence of real numbers

1.1 Sequence and subsequence

1.1.1 Definition and examples.

1.1.2 Limit of sequence and examples using definition.

1.1.3 Theorem: If $\{S_n\}_{n=1}^{\infty}$ is sequence of non-negative real numbers and if $\lim S_n = L$ then L $\geq$ 0.

1.1.4 Convergent sequences and examples.

1.1.5 Theorem: If the sequence of real numbers $\{S_n\}_{n=1}^{\infty}$ is convergent to L, then $\{S_n\}_{n=1}^{\infty}$ can not converge to limit distinct from L.

1.1.6 Theorem (without proof) : If the sequence of real numbers $\{S_n\}_{n=1}^{\infty}$ is convergent to L, then any subsequence of $\{S_n\}_{n=1}^{\infty}$ is also convergent to L.

1.1.7 Theorem (without proof): All subsequences of a convergent sequence of realnumbers converge to the same limit.

1.1.8 Bounded sequences and examples.

1.1.9 Theorem: If the sequence of real numbers $\{S_n\}_{n=1}^{\infty}$ is convergent, then it isbounded.

1.2 Monotone Sequences

1.2.1 Definition and examples.

1.2.2 Theorem: A non-decreasing sequence which is bounded above is convergent.

1.2.3 Theorem: A non-increasing sequence which is bounded below is convergent.

1.2.4 Corollary**:** The sequence $\{(1 + 1/n)^n\}$ is convergent.

1.2.5 Theorem (without proof): A non-decreasing sequence which is not boundedabove diverges to infinity.

1.2.6 Theorem (without proof)**:** A non-increasing sequence which is not boundedbelow diverges to minus infinity.

1.2.7 Theorem : Abounded sequence of real numbers has convergentsubsequence.

1.3 Operations on convergent sequences

1.3.1 Theorem**:** If $\{S_n\}_{n=1}^{\infty}$ and $\{t_n\}_{n=1}^{\infty}$ are sequences of real numbers, if lim S_n=L and lim t_n = M then lim ($S_n + t_n$) = L + M.

1.3.2 Theorem**:** If $\{S_n\}_{n=1}^{\infty}$ and $\{t_n\}_{n=1}^{\infty}$ are sequences of real numbers, if lim S_n=L and lim t_n= M then lim ($S_n - t_n$) = L – M.

1.3.3 Theorem**:** If $\{S_n\}_{n=1}^{\infty}$ is sequence of real numbers, if c $\in R$ and if lim S_n= L. then lim c$S_n = cL$.

1.3.4 Theorem: If 0< x < 1 , then the sequence $\{ x^n \}$ converges to 0.

1.3.5 Lemma: If $\{S_n\}_{n=1}^{\infty}$ is sequence of real numbers which converges to L then $\{S_n 2\}_{n=1}^{\infty}$ converges to L^2.

1.3.6 Theorem**:** If $\{S_n\}_{n=1}^{\infty}$ and $\{t_n\}_{n=1}^{\infty}$ are sequences of real numbers, if lim S_n=L and lim t_n= M then lim ($S_n . t_n$) = LM.

1.3.7 Theorem**:** If $\{S_n\}_{n=1}^{\infty}$ and $\{t_n\}_{n=1}^{\infty}$ are sequences of real numbers, if lim

S_n=L and lim t_n= M then lim (S_n / t_n) = L / M.

1.4 Limit Superior and Limit Inferior of Sequences

1.4.1 Definition and examples.

1.4.2 Theorem: If $\{S_n\}_{n=1}^{\infty}$ is convergent sequence of real numbers, then $\lim_{n\to\infty} sup\, S_n = \lim_{n\to\infty} S_n$.

1.4.3 Theorem: If $\{S_n\}_{n=1}^{\infty}$ is convergent sequence of real numbers, then $\lim_{n\to\infty} inf\, S_n = \lim_{n\to\infty} S_n$.

1.4.4 Theorem: If $\{S_n\}_{n=1}^{\infty}$ is a sequences of real numbers, and if $\lim_{n\to\infty} sup\, S_n = \lim_{n\to\infty} inf\, S_n$= L where L $\in$ R, then $\{S_n\}_{n=1}^{\infty}$ is convergent and $\lim_{n\to\infty} S_n$ = L.

1.4.5 Theorem: If $\{S_n\}_{n=1}^{\infty}$ and $\{t_n\}_{n=1}^{\infty}$ are bounded sequences of real numbersand if $S_n \le t_n$ then i) $\lim_{n\to\infty} \sup S_n \le \lim_{n\to\infty} \sup t_n$.

ii) $\lim_{n\to\infty} \inf S_n \le \lim_{n\to\infty} \inf t_n$

1.4.6 Theorem: If $\{S_n\}_{n=1}^{\infty}$ and $\{t_n\}_{n=1}^{\infty}$ are bounded sequences of real numbers then i) $\lim_{n\to\infty} \sup (S_n+t_n) \le \lim_{n\to\infty} \sup S_n + \lim_{n\to\infty} \sup t_n$.

ii) $\lim_{n\to\infty} \inf (S_n+t_n) \ge \lim_{n\to\infty} \inf S_n + \lim_{n\to\infty} \inf t_n$.

1.5 The Cauchy Sequence

1.5.1 Definition and examples

1.5.2 Theorem: If the sequence of real numbers $\{S_n\}_{n=1}^{\infty}$ converges, then $\{S_n\}_{n=1}^{\infty}$ is Cauchy sequence.

1.5.3 Theorem: If $\{S_n\}_{n=1}^{\infty}$ is the Cauchy sequence of real numbers then $\{S_n\}_{n=1}^{\infty}$ is bounded.

1.5.4 Theorem: If $\{S_n\}_{n=1}^{\infty}$ is the Cauchy sequence of real numbers then $\{S_n\}_{n=1}^{\infty}$ is convergent.

1.5.5 Definition and examples of (C , 1) summability of sequence.

UNIT 2 Infinite Series

2.1 Convergent and Divergent Series

2.1.1 Definition**:** Infinite series, convergent and divergent series, sequence of partialsum of series and examples.

2.1.2 A necessary condition for convergence**:** A necessary condition forconvergence of an infinite series $\sum u_n$ is that lim u_n= 0.

2.1.3 Cauchy's General Principal of Convergence (statement only).

2.1.3 Theorem**:** A series $\sum u_n$ converges iff for every $\in$ $\epsilon > 0$ there exists a positive number m such that $|u_{n+1} + u_{n+2} + \quad + u_{n+p}| < \epsilon$, for every all n $\ge$ m and p $\ge$

2.2 Positive Term Series

2.2.1 Definition and examples.

2.2.2 Theorem**:** A positive term series converges iff its sequence of partial sums isbounded above.

2.2.3 Geometric Series**:** The positive term geometric series $\sum_{n=0}^{\infty} r^n$ converges for $r < 1$, and diverges to infinity for $r \geq 1$.

2.2.4 Theorem**:** A positive term series $\sum_{n=1}^{\infty} 1/n^p$ is convergent if and only if p > 1.

2.3 Comparison Tests For Positive Term Series

2.2.5 Comparison Test (First Type)**):** If $\sum u_n$ and $\sum v_n$ are two positive term series≠ and k ≠ 0, a fixed positive real number (independent of n) and there exists apositive integer m≤such that $u_n \geq kv_n$, for every n m,then

(a) $\sum u_n$ is convergent , if $\sum v_n$ is convergent, and

(b) $\sum v_n$ is divergent, if $\sum u_n$ is divergent.

2.2.6 Examples.

2.2.7 Limit Form**:** If $\sum u_n$ and $\sum v_n$ are two positive term series such that lim (u_n / v_n) = L, where L is a non zero finite number, then the two series converge or diverge together.

2.2.8 Comparison Test (Second Type): If $\sum u_n$and $\sum v_n$ are two positive termseries, and there exists a positive number m such that

$(u_n / u_{n+1}) \geq (v_n / v_{n+1})$, for every n ≥ m, then (a) $\sum u_n$ is convergent, if $\sum v_n$ is convergent, and (b) $\sum v_n$ is divergent , if $\sum u_n$ is divergent.

2.2.9 Examples.

2.2.10 Cauchy's Root Test: If $\sum u_n$ is a positive term series such that lim $(u_n)^{1/n}$ = L, then the series (i) converges, if L < 1, (ii) diverges, if L > 1,and (iii) the test fails to give any definite information, if L= 1.

2.3.7 Examples.

2.3.8 D'Alembert's Ratio Test**:** If $\sum u_n$ is a positive term series, such that lim (u_{n+1} / u_n) = L, then the Series (i) converges, if L < 1. (ii) diverges, if L > 1, and (iii) the test fails, if L = 1.

2.3.9 Examples.

2.3.10 Raabe's Test**:** If $\sum u_n$ is a positive term series such that Lim n{ (u_n / u_{n+1}) – 1 } = L, then the series (i) converges, if L >1. (ii) diverges, if L < 1, and (iii) the test fails, if L = 1.

2.3.11 Examples**.**

2.4 Alternating Series

2.4.1 Definition and examples.

2.4.2 Leibnitz Test**:** If the alternating series $u_1 - u_2 + u_3 - u_4 +$, ($u_n > 0$, for every n) is such that (i) $u_{n+1} \leq u_n$, for every n and (ii) lim $u_n = 0$, then theseries converges.

2.4.3 Examples.

2.5 Absolute and Conditional Convergence

2.5.1 Definition and examples .

2.5.2 Theorem**:** Every absolutely convergent series is convergent.

2.5.3 Examples.

Recommended Books:

1. **R.R.Goldberg**, Methods of Real Analysis, Oxford & IBH Publishing Co. Pvt.Ltd., New Delhi. **For Unit 1**
2. **S.C.Malik and SavitaArora**, Mathematical Analysis (Fifth Edition), New Age International (P) Limited, 2017. **For Unit 2**

Reference Books:

1. **R.G.Bartle and D.R.Sherbert**, Introduction to Real Analysis, Wiley India Pvt.Ltd., Fourth Edition 2016.
2. **D Somasundaram and B Choudhary**, First Course in Mathematical Analysis,Narosa

Publishing House New Delhi, Eighth Reprint 2013.

3. **P.K.Jain and S.K.Kaushik,** An Introduction to Real Analysis, S.Chand&Company Ltd. New Delhi, First Edition 2000.
4. **Shanti Narayan and M.D.Raisinghania,** Elements of Real Analysis, S.Chand&Company Ltd. New Delhi, Fifteenth Revised Edition 2014
5. **Shanti Narayan and P.K.Mittal,** A Course of Mathematical Analysis, S.Chand& Company Ltd. New Delhi, Reprint 2016

Title of Course: Algebra-II

Unit - 1 **Groups**

Lagrange's theorem and its Consequences

1.1.1 **Definition** of Index of a subgroup

1.1.2 **Theorem(Lagrange):** If G is any finite group and H is any subgroup of G, then O(H) divides O(G).

1.1.3 **Corollary:** The index of any subgroup of a finite group is a divisor of the order of the group.

1.1.4 **Corollary:** If G is a finite group and a $\in$ G, then O(a) divides O(G).

1.1.5 **Corollary:** If G is a finite group of order n then for all a $\in$ G, $a^n = e$, where e is the identity element of G.

1.1.6 **Theorem(Euler's theorem):** If n is any positive integer and a is relatively prime to n, then $a^{\phi(n)} \equiv 1 (\text{mod } n)$

1.1.7 **Theorem(Fermat's theorem):** If a is any integer and p is any positive prime, then $a^p \equiv a (\text{mod } p)$.

1.2 Normal subgroups and its Properties

1.2.1 **Definition** of Normal subgroup and examples

1.2.2 **Theorem:** A subgroup H of a group G is normal if and only if $gHg^{-1} = H$ for all g in G.

1.2.3 **Theorem:** A subgroup H of a group G is normal if and only if every right coset of H in G is a left coset of H in G.

1.2.4 **Corollary:** Every subgroup of an abelian group is a normal subgroup.

1.2.5 **Theorem:** A subgroup H of a group G is normal in G if and only if the product of any two right (or left) cosets H in G is again a right (or left) coset of H in G.

1.2.6 Results related to Normal subgroups

(i) The intersection of any two normal subgroups of a group is also a normal subgroup.

(ii) The product of any two normal subgroups of a group is a subgroup of the group.

(iii) Let H be a subgroup and K be normal subgroup of the group G. Then HK is normal in H.

(iv) If N is a normal subgroup of G and H is any subgroup of G, then NH is a subgroup of G.

(v) The center Z of a group G is a normal subgroup of G.

(vi) The center Z of a group is a normal subgroup of a normalizer of an element.

1.3 Factor Group (Quotient Group)

1.3.1 **Definition** of Factor Group or Quotient Group and examples

1.3.2 **Theorem:** The set G/H = {Ha | a $\in$ G} of all cosets of a normal subgroup H, of the group G, is a group G, is a group under the binary operation defined by Ha .Hb = Hab, for all Ha, Hb $\in$ G/H.

1.3.3 **Theorem:** If H is a normal subgroup of finite order, then O(G/H) = O(G)/O(H).

1.3.4 **Theorem:** Every Quotient group of an abelian group is abelian.

1.3.5 **Theorem:** Every factor group of a cyclic group is cyclic

1.4 Homomorphism of Groups

1.4.1 **Definition** of Homomorphism, Isomorphism, Automorphism and Endomorphism of Groups and examples.

1.4.2 **Theorem:** Let f : G → G’ be a homomorphism from the group (G, .) into the group (G’, *). Then

(i) f(e) = e’, where e and e’ are the identity elements of the groups G and G’ respectively.

(ii) $f(a^{-1}) = [f(a)]^{-1}$, for all a ∈ G.

1.4.3 **Theorem:** If f is a homomorphism of a group G into a group G’, then the range f(G) = {f(g) | for all g ∈ G} is a subgroup of G’.

1.4.4 **Theorem:** The homomorphic image of the group G in the group G’ is a subgroup of G’.

1.4.5 **Theorem:** Let f : G → G’ be a homomorphism from the group G into the group G’ and H is a subgroup of G, then f(H) is also a subgroup of G’.

1.4.6 **Theorem:** Let $f: G \to G$ be a homomorphism of the group G into itself and H is a cyclic subgroup of G, then f(H) is again a cyclic subgroup of G.

Unit – 2 Normal subgroups

2.1. Kernel of a Homomorphism

2.1.1. **Definition** of Kernel of a Homomorphism and examples.

2.1.2. **Theorem:** Let f : G → G’ be a homomorphism of a group G into G’ with Kernel K. Then K is a normal subgroup of G.

2.1.3. **Theorem:** Let → $f: G \to G'$ be a homomorphism of a group G into G’ with Kernel K. Then f is one – one if and only if K = {e}, where e is the identity element of G.

2.1.4. **Corollary:** A homomorphism f from the group G onto the group G’ is an isomorphism if and only if Ker f = {e}.

2.1.5. **Theorem:** Let G be a group and H be a normal subgroup of G. Then G/H is a homomorphic image of G with H as its Kernel.

2.1.6. **Theorem (Fundamental Homomorphism Theorem):** Let f be a homomorphism of a group G into a group G’, with kernel K. Then f(G) is isomorphic to factor group G/K.

2.1.7. Results related to Isomorphism

(i) If $f: G \to G'$ be an isomorphism of a group G onto a group G' and a is any element of G then the order of f(a) equals the order of a.

(ii) If $f: G \to G'$ be an isomorphism and G is an abelian group then G’ is also abelian.

(iii) Any infinite cyclic group is isomorphic to the group Z of integers, under addition.

(iv) Any finite cyclic group of order n is isomorphic to additive group of integers modulo n.

2.2. Permutation Group

2.2.1. **Definition** of Permutation, Degree of permutation, Equality of two permutations, Identity permutations, Inverse and Composition of permutation and Symmetric group and examples.

2.2.2. **Theorem:** Let S be a non - empty finite set of n elements. The set S_n of all permutations of degree n defined on S, is a finite group of order n!, under the permutation multiplication.

2.2.3. **Theorem (Cayley's Theorem):** Every finite group is isomorphic to a group of permutation.

2.3. Rings

2.3.1. Definition and examples.
2.3.2. Basic Properties.
2.3.3. Homomorphism and isomorphism in a ring.
2.3.4. Multiplicative questions: Fields
2.3.5. Examples of Commutative and non-commutative rings.
2.3.6. Rings from number system, Z_n the ring of integers modulo n.

2.4. Subrings

2.4.1. Definition and examples.
2.4.2. Basic properties
2.4.3. Ideals: Definition and examples.
2.4.4. Examples of subring which are not ideals.

Recommended Books:

1 **J. B. Fraleigh**, A First Course in Abstract Algebra, Narosa Publishing House New Delhi, Tenth Reprint 2003.

2 **V. K. Khanna and S. K. Bhambri**, A Course in Abstract Algebra, Vikas Publishing House Pvt Ltd., New Delhi, Fifth Edition 2016.

Reference Books:

1 **I.N. Herstein**, Topics in Algebra, Wiley india Pvt. Ltd,

2 **M. Artin**, Algebra, Prentice Hall of India, New Delhi, 1994

3 **N. S. Gopalkrishnan,** University Algebra, New Age International New Delhi, Second Edition 1986

Syllabus For
B.Sc. Part -I (Mathematics)
SEMESTER I AND II

Title of the Couse: Differential Calculus

Unit – 1:- Hyperbolic Functions

1.1 De- Moivre's Theorem. Examples.

1.2 Applications of De- Moivre's Theorem , n^{th} roots of unity

1.3 Hyperbolic functions. Properties of hyperbolic functions.

1.4 Differentiation of hyperbolic functions

1.5 Inverse hyperbolic functions and their derivatives. Examples

1.6 Relations between hyperbolic and circular functions.

1.7 Representation of curves in Parametric and Polar co-ordinates.

Unit – 2: - Higher Order Derivatives

2.1 Successive Differentiation

n^{th} order derivative of standard functions: $(ax+b)^m$, e^{ax}, a^{mx}, $1/(ax+b)$, $\sin(ax+b)$,

$\cos(ax+b)$, $e^{ax}\sin(ax+b)$, $e^{ax}\cos(ax+b)$.

2.2 Leibnitz's Theorem (with proof).

2.3 Partial differentiation, Chain rule (without proof) and its examples.

2.4 Euler's theorem on homogenous functions.

2.5 Maxima and Minima for functions of two variables.

2.6 Lagrange's Method of undetermined multipliers.

Recommended Books:

(1) H. Anton, I. Birens and Davis, **Calculus**, John Wiley and Sons, Inc.2002.
(2) G. B. Thomas and R. L. Finney, **Calculus and Analytical Geometry**, Pearson Education, 2007.
(3) Maity and Ghosh, **Differential Calculus**, New Central Book Agency (P) limited, Kolkata, India. 2007.

Reference Books:

(1) Shanti Narayana and P. K. Mittal, **A Course of mathematical Analysis**, S. Chand and Company, New Delhi. 2004.
(2) S. C . Malik and Savita arora, **Mathematical Analysis (second Edition)**, New Age International Pvt. Ltd., New Delhi, Pune, Chennai.

Title of the Course: Calculus

Unit – 1: - Mean Value Theorems and Indeterminate Forms

1.1 Rolle's Theorem

1.2 Geometrical interpretation of Rolle's Theorem.

1.3 Examples on Rolle's Theorem

1.4 Lagrange's Mean Value Theorem (LMVT)

1.5 Geometrical interpretation of LMVT.

1.6 Examples on LMVT

1.7 Cauchy's Mean Value Theorem (CMVT)

1.8 Examples on CMVT

1.9 Taylor's Theorem with Lagrange's and Cauchy's form of remainder (without proof)

1.10 Maclarin's Theorem with Lagrange's and Cauchy's form of remainder (without proof)

1.11 Maclarin's series for sin x, cos x, e^x, log (1+x), $(1+x)^m$.

1.12 Examples on Maclarin's series

1.13 Indeterminate Forms

1.14 L'Hospital Rule, the form $\frac{0}{0}$, $\frac{\infty}{\infty}$, and Examples.

1.15 L'Hospital Rule, the form $0 \times \infty$, $\infty - \infty$. and Examples.

1.16 L'Hospital Rule, the form 0^0 , ∞^0 , 1^∞ . and Examples.

Unit 2: - Limits and Continuity of Real Valued Functions

2.1 $\in$ - δ definition of limit of function of one variable, Left hand side limits and Right hand side limits .

2.2 Theorems on Limits (Statements Only)

2.3 Continuous Functions and Their Properties

2.3.1 If f and g are two real valued functions of a real variable which are continuous at x = c then (i) f + g (ii) f – g (iii) f.g are continuous at x = c. and (iv) f/g is continuous at x = c , g(c) $\neq$ 0.

2.3.2 Composite function of two continuous functions is continuous.

2.4 Classification of discontinuities (First and second kind).

2.4.1Types of Discontinuities :(i) Removable discontinuity(ii) Jump discontinuity of first kind
(iii) Jump discontinuity of second kind

2.5 Differentiability at a point, Left hand derivative, Right hand derivative, Differentiability in the interval [a,b].

2.6 Theorem: Continuity is necessary but not a sufficient condition for the existence of a derivative.

2.7.1. If a function f is continuous in a closed interval [a, b] then it is bounded in [a, b].

2.7.2.If a function f is continuous in a closed interval [a, b] then it attains its bounds at least once in [a, b].

2.7.3.If a function f is continuous in a closed interval [a, b] and if f(a), f(b) are of opposite signs then there exists $c \in [a, b]$ such that $f(c) = 0$. (Statement Only)

2.7.4. If a function f is continuous in a closed interval [a, b] and if $f(a) \neq f(b)$ then f assumes every value between f(a) and f(b). (Statement Only)

Recommended Books:

(4) H. Anton, I. Birens and Davis, **Calculus**, John Wiley and Sons, Inc.2002.

(5) G. B. Thomas and R. L. Finney, **Calculus and Analytical Geometry**, Pearson Education, 2007.

(6) Maity and Ghosh, **Differential Calculus**, New Central Book Agency (P) limited, Kolkata, India. 2007.

Reference Books:

(3) Shanti Narayana and P. K. Mittal, **A Course of mathematical Analysis**, S. Chand and Company, New Delhi. 2004.

(4) S. C . Malik and Savita arora, **Mathematical Analysis (second Edition)**, New Age International Pvt. Ltd., New Delhi, Pune, Chennai.

Title of the Course: Differential Equations

Unit 1: Differential Equations of First Order

1.1: Differential Equations of First Order and First Degree.

1.1.1 : Exact Differential Equations.

1.1.2 : Necessary and Sufficient condition for exactness.

1.1.3 : Working Rule for solving an Exact Differential Equation.

1.1.4: Integrating Factor.

1.1.5 : Integrating Factor by Inspection and examples.

1.1.6 : Integrating Factor by using Rules (Without Proof) and Examples.

1.1.7 : Linear Differential Equations: Definition, Method of Solution and examples.
1.1.8: Bernoulli's Equation: Definition, Method of Solution and Examples.

1.2: Differential Equations of First Order but Not of First Degree:

1.2.1: Introduction.

1.2.2: Equations solvable for p: Method and Problems.

1.2.3: Equations solvable for x: Method and Problems.

1.2.4: Equations solvable for y: Method and Problems.

1.2.5: Clairaut's Form: Method and Problems.

1.2.6: Equations Reducible to Clairaut's Form.

Unit 2: Linear Differential Equations

2.1: Linear Differential Equations with Constant Cofficients

2.1.1 : Introduction and General Solution.

2.1.2 : Determination of Complementary Function

2.1.3: The Symbolic Function 1/f(D):Definition.

2.1.4: Determination of Particular Integral.

2.1.5 : General Method of Particular Integral.

2.1.6 : Theorem: $\frac{1}{(D-a)^n}e^{ax} = \frac{x^n}{n!}e^{ax}$, where n is a positive integer.

2.1.7 : Short Methods of Finding P.I. when X is in the form e^{ax}, sin ax , cos ax ,

x^m(m being a positive integer), e^{ax}V , x V where V is a function of x.

2.1.8 : Examples.

2.2: Homogeneous Linear Differential Equations (The Cauchy-Euler Equations)

2.2.1: Introduction and Method of Solution.

2.2.2 : Legendre's Linear Equations.

2.2.3 : Method of Solution of Legendre's Linear Equations.

2.2.4: Examples.

Recommended Books:

(1) M. D. Raisinghania, Ordinary and Partial Differential Equations, Eighteenth Revised Eition 2016; S. Chand and Company Pvt. Ltd. New Delhi

(2) Shepley L. Ross, Differential Equations, Third Edition 1984; John Wiley and Sons, New York

Reference Books:

(1) R. K. Ghosh and K. C, Maity, An Introduction to Differential Equations, Seventh Edition, 2000; Book and Allied (P) Ltd

(2) D. A. Murray, Introductory course in DIfferential Equations, Khosala Publishing House, Delhi.

Title of the Course: **Higher Order Ordinary Differential Equations And Partial Differential Equations**

Unit 1: Second Order Linear Differential Equations and Simultaneous Differential Equations

:

Second Order Linear Differential Equations

1.1.1: The General Form.

1.1.2: Complete Solution when one Integral is known: Method and Examples.

1.1.3: Transformation of the Equation by changing the dependent variable

(Removal of First order Derivative).

1.1.4: Transformation of the Equation by changing the independent variable.

1.1.5: Method of Variation of Parameters.

1.1.6: Examples.

1.1 Ordinary Simultaneous Differential Equations and Total Differential Equations

1.2.1: Simultaneous Linear Differential Equations of the Form $\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$

1.2.2: Methods of Solving Simultaneous Linear Differential Equations.

1.2.3: Total differential equations Pdx + Qdy + Rdz = 0

1.2.4: Neccessary condition for Integrability of total differential equation

1.2.5: The condition for exactness.

1.2.6 : Methods of solving total differential equations:

a) Method of Inspection

b) One variable regarding as a constant

1.2.7 : Geometrical Interpretation of Ordinary Simultaneous Differential Equations

1.2.8: Geometrical Interpretation of Total Differential Equations

1.2.9: Geometrical Relation between Total Differential equations and Simultaneous differential Equations.

Unit 2 : Partial Differential Equations

2.1: Partial Differential Equations

2.1.1 : Introduction

2.1.2 : Order and Degree of Partial Differential Equations

2.1.3: Linear and non-linear Partial Differential Equations

2.1.4 : Classification of first order Partial Differential Equations

2.1.5 : Formation of Partial Differential Equations by the elimination of arbitrary constants

2.1.6: Formation of Partial Differential Equations by the elimination of arbitrary functions Ø from the equation Ø(u,v) = 0 where u and v are functions of x, y and z.

2.1.7: Examples.

2.2 : First Order Partial Differential Equations

2.2.1 : First Order Linear Partial Differential Equations

2.2.2: Lagrange's equations Pp + Qq = R

2.2.3: Lagrange's methods of solving Pp + Qq = R

2.2.4: Examples

2.3 : Charpit's method

2.3.1 : Special methods of solutions applicable to certain standard forms

2.3.2: Only p and q present

2.3.3: Clairaut's equations

2.3.4: Only p, q and z present

2.3.5: f(x,p) = g(y,q)

2.3.6: Examples

Recommended Books:

(1) M. D. Raisinghania, Ordinary and Partial Differential Equations, Eighteenth Revised Eition 2016; S. Chand and Company Pvt. Ltd. New Delhi

(2) Shepley L. Ross, Differential Equations, Third Edition 1984; John Wiley and Sons, New York

(3) Ian Sneddon, Elements of Partial Differential Equations, Seventeenth Edition, 1982; Mc-Graw-Hill International Book Company, Auckland

Reference Books:

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