ENT - 17

Seat No.	

Total No. of Pages : 20

P.G. Entrance Examination, July - 2023
M.Sc. MATHEMATICS/M.Sc. (MATHEMATICS WITH COMPUTER APPLICATION)
Sub. Code: 58716

Day and Date : Tuesday, 18- 07 - 2023 **Total Marks : 100**
Time : 1.00 p.m. to 2.30 p.m.

Instructions :
1) **All questions are compulsory.**
2) **Each question carries 1 mark.**
3) **Answers should be marked in the given OMR answer sheet by darkening the appropriate option.**
4) **Follow the instructions given on OMR Sheet.**
5) **Rough work shall be done on the sheet provided at the end of question paper.**

1) Let H_1, H_2 are subgroups of a finite group G. Consider the following statements.

(I) $H_1 \cup H_2$ is always a subgroup of G.

(II) $O(H_2)$ divides O(G). Then

A) Both (I) and (II) are true B) Both (I) and (II) are false

C) Only (II) is true D) Only (I) is true

2) If G be a group such that O(G) = p, where p be a prime number. Then G will have only _____ subgroups.

A) 2 B) $p + 1$

C) 1 D) $p!$

3) Let G be a cyclic group and let H be a subgroup of G. Consider the following statements.

(I) H is cyclic.

(II) H is not abelian. Then

A) Both (I) and (II) are true B) Both (I) and (II) are false

C) Only (II) is true D) Only (I) is true

4) Let G be a simple non abelian group. Consider the following statements.

(I) $G = G'$.

(II) G has only two normal subgroups. Then

A) Both (I) and (II) are true
B) Both (I) and (II) are false
C) Only (II) is true
D) Only (I) is true

5) Consider the following statements:

(I) $< Z, +; >$ is an integral domain.

(II) $< Z, +; >$ is not a field. Then

A) Both (I) and (II) are true
B) Both (I) and (II) are false
C) Only (I) is true
D) Only (II) is true

6) Characteristic of a non-zero integral domain is _____.

A) 10
B) 0 or *p* (a prime number)
C) 1
D) ∞

7) If A and B be two ideals of a ring R. Consider the following statements:

(I) A + B is an ideal of R.

(II) $A \subseteq A+B$ and $B \subseteq A+B$. Then

A) Both (I) and (II) are true
B) Both (I) and (II) are false
C) Only (I) is true
D) Only (II) is true

8) If $f : R \to R'$ be a ring homomorphism, then Kernel of *f* is Ker *f* = _____.

A) $\{x \in R' \mid f(x) = 0\}$ where 0 is zero of R

B) $\{0\}$

C) $\{x \in R' \mid f^{-1}(x) = 0\}$ where 0 is zero of R

D) $\{x \in R \mid f(x) = 0'\}$ where $0'$ is zero of R'

9) Let R and R' be two rings. A _____ from R to R' is called an imbedding mapping.

A) one-one
B) homomorphism
C) one-one homomorphism
D) onto homomorphism

10) An ideal $M \neq R$ of a ring R is said to be maximal ideal of R if whenever N is an ideal of R such that _____ then either N = M or N = R.

A) $M \subseteq R \subseteq N$
B) $N \subseteq R \subseteq M$
C) $N \subseteq M \subseteq R$
D) $M \subseteq N \subseteq R$

11) Let < X, *d* > and $< Y, \rho >$ be two metric spaces and $f : X \to Y$ is function. Then *f* is continuous if and only if $f^1(G)$ is open in X whenever _____.

A) G is closed in X
B) G is open in X
C) G is neither open nor closed
D) G is open in Y

12) Let < X, *d* > and $< Y, \rho >$ be two metric spaces and $f : X \to Y$ be a function. Then *f* is continuous if and only if $\overline{f^{-1}(B)} \subset f^{-1}(\overline{B})$ for every _____.

A) set B of X
B) subset B of X
C) set B of Y
D) subset B of Y

13) If *f* is continuous at a and if $c \in R$ then *cf* is continuous

A) on R
B) at *a*
C) at *c*
D) at *a.c*

14) If the real valued functions *f* and *g* are continuous at $a \in R$ then which of the following statement is false?

A) $f + g$ is always continuous at *a*
B) $f - g$ is always continuous at *a*
C) $f \cdot g$ is always continuous at *a*
D) $\frac{f}{g}$ is always continuous at *a*

15) If $a \in R_d$ then $\{a\}$ is _____.

A) B[a;1]
B) open set in R_d
C) not open set in R_d
D) not closed set in R_d

16) Every subset of R_d is _____.

A) both open and closed
B) only open
C) only closed
D) neither open nor closed

17) If E is any subset of metric space M then which of the following statement is true?

A) $E \subset \overline{E}$
B) E is closed subset of M if $E = \overline{E}$
C) E is closed and $\overline{E} = \overline{\overline{E}}$
D) All the statements in (A), (B) and (C) are true

18) The intersection of any number of closed sets is always

A) closed
B) open
C) open and closed
D) neither open nor closed

19) The set in metric space X is open if and only if its complement is _____.

A) closed
B) open
C) always empty set
D) always X

20) If f and g are continuous functions from a metric space M_1 into a metric space M_2 then which of the following statement is false?

A) $f + g$ is always continuous on M_1
B) $f - g$ is always continuous on M_1
C) $f \cdot g$ is always continuous on M_1
D) $\frac{f}{g}$ is always continuous on M_1

21) Sequencing problems involving of two jobs on '*n*' machines _____.

A) Can be solved graphically

B) Can not be solved graphically

C) Have a condition that the processing of two jobs must be in the same order

D) None of the above

22) In 2 machines and 5 job problem, the processing times are given the following table:

Job	1	2	3	4	5
A	5	1	9	3	10
B	2	6	7	8	4

The optimum sequence would be : _____.

A) 2 - 1 - 5 - 4 -3

B) 2 - 4 - 3 - 5 -1

C) 2 - 5 - 4 - 3 - 1

D) 2 - 5 - 1 - 4 - 3

23) If A_i, B_i and C_i denote the processing times of i^{th} job on three machines A, B and C respectively, then a *n* - job three machine problem can be reduced to an *n* - job and two machine problem, provide that:

A) $\text{Min}\, A_i \geq \text{Max}\, B_i$ and / or $\text{Min}\, C_i \leq \text{Max}\, B_i$

B) $\text{Min}\, A_i \leq \text{Max}\, B_i$ and / or $\text{Min}\, C_i \geq \text{Max}\, B_i$

C) $\text{Min}\, A_i \geq \text{Max}\, B_i$ and / or $\text{Min}\, C_i \geq \text{Max}\, B_i$

D) $\text{Min}\, A_i \leq \text{Max}\, B_i$ and / or $\text{Min}\, C_i \leq \text{Max}\, B_i$

24) The size of pay off matrix of a game can be reduced by using the principle of _____.

A) Game inversion

B) Rotation reduction

C) Dominance

D) Game transpose

25) When maximim and minimax values of the game are same, then _____.

A) There is a saddle point

B) Solution does not exists

C) Strategies are mixed

D) None of the above

26) Find the value of the game $\begin{bmatrix} 10 & 6 \\ 8 & 2 \end{bmatrix}$.

A) 2 B) 6
C) 10 D) 8

27) When total supply is equal to total demand in a transportation problem, the problem is said to be _____.

A) Balanced B) Unbalanced
C) Degenerate D) None of these

28) In a 5 × 7 transportation problem, degeneracy would arise if the number of filled slots were _____.

A) equal to 35 B) equal to 34
C) more than 12 D) less than 11

29) In Vogel's approximation method _____.

A) The cost difference indicate the penalties for not using the respective least cost routes
B) Initial solution to transportation problem is not applicable, if some routes are prohibited
C) Degeneracy never occurs
D) None of the above

30) In an assignment problem involving four workers and three jobs, total number of assignments possible are _____.

A) 4 B) 3
C) 7 D) 12

31) Laplace transform of e^{t^3} = _____.

A) $\frac{1}{s-3}$ B) $\frac{3}{s-1}$
C) $\frac{1}{s+3}$ D) does not exist

32) The value of $\int_0^{\infty} e^{-2t} \cos 3t \; dt =$ _____.

A) $\frac{2}{13}$ B) $\frac{1}{13}$

C) 0 D) $\frac{2}{3}$

33) The Laplace transform of unit step function $\mathrm{H}(t-a)$ is _____.

A) $\frac{e^{-2as}}{s}$ B) $\frac{e^{-s}}{s}$

C) $\frac{e^{-as}}{s^2}$ D) $\frac{e^{-as}}{s}$

34) Laplace transform of $\int_0^t \int_0^t \text{coshat} \; dt \; dt =$ _____.

A) $\frac{s}{s^2 - a^2}$ B) $\frac{1}{s\left(s^2 - a^2\right)}$

C) $\frac{s}{s^2 + a^2}$ D) $\frac{1}{s\left(s^2 + a^2\right)}$

35) $\mathrm{L}^{-1}\left\{\frac{1}{s^{101}}\right\} =$ _____.

A) $\frac{t^{100}}{101!}$ B) $\frac{t^{101}}{101!}$

C) $\frac{t^{101}}{100!}$ D) $\frac{t^{100}}{100!}$

36) $L^{-1}\{f(s-a)\} =$ _____ where $L\{f(t)\} = f(s)$.

A) $e^{at} f(at)$ B) $e^{at} f(t)$

C) $e^{t} f(at)$ D) $e^{at} f(2t)$

37) $L^{-1}\left\{\frac{1}{\sqrt{s-5}}\right\} =$ _____.

A) $e^{+5t}\frac{1}{\sqrt{t}}$ B) $e^{+5t}\frac{1}{\sqrt{\pi t}}$

C) $e^{t}\frac{5}{\sqrt{\pi t}}$ D) $e^{-5t}\frac{1}{\sqrt{\pi t}}$

38) If $f_1(t) = \frac{1}{t}, f_2(t) = \frac{1}{\sqrt{t}}$ and $s > 0$ then Laplace transform of

A) $f_1(t)$ does not exit but of $f_2(t)$ exits.

B) both $f_1(t)$ and $f_2(t)$ exit.

C) both $f_1(t)$ and $f_2(t)$ do not exit.

D) $f_2(t)$ does not exit but of $f_1(t)$ exits.

39) If $f(s)$ is Fourier transform of $F(x)$ then Fourier transform of $F(x)\cdot \cos ax$ is

A) $\frac{1}{2}[f(s-a) - f(s+a)]$ B) $\frac{1}{2}[f(s-a) + f(s+a)]$

C) $\frac{1}{2}[f(s) + f(s+a)]$ D) $\frac{1}{2}[f(s-a) + f(s)]$

40) $L\left\{\frac{1-e^{3t}}{t}\right\} =$ _____.

A) $-\log\frac{s-3}{s}$ B) $\log\frac{s+3}{s}$

C) $\log\frac{s}{s-3}$ D) $\log\frac{s-3}{s}$

41) Which of the following function of a complex variable z is not differentiable at $z = 0$?

A) z^2 B) $\overline{z}\cos z$

C) $e^z + \sin z$ D) $|z|^2$

42) If $f(z) = u(x,y) + iv(x,y)$ is such that both $u(x,y)$ and $v(x,y)$ satisfies Cauchy Riemann equations and if $u_x(x,y) = x$ and $u_y(x,y) = -y$ then for any $z = (x,y)$, the value of $f'(z) =$ _____.

A) z B) $-z$

C) 0 D) $\overline{z}$

43) e^z is a periodic function with period _____.

A) πi B) $2\pi i$

C) π D) 2π

44) $\int_0^1 (1-it)^2\, dt =$ _____.

A) $\frac{2}{3} - i$ B) $\frac{2}{3} + i$

C) $\frac{4}{3} - i$ D) $\frac{4}{3} + i$

45) Isolated singularities of $f(z) = \frac{z^2+4}{(z+6)(z^2-4)}$ are _____.

A) $z = 2, -2, 6$ B) $z = 2i, -2i, -6$

C) $z = 2i, -2i, 6$ D) $z = 2, -2, -6$

46) The sequence of complex numbers $z_n = -1 + i\frac{(-1)^n}{n}, n = 1, 2, \ldots.$ is _____.

A) divergent B) converges to –1

C) converges to 0 D) oscillatory

47) If $f(z)=\sum_{n=0}^{\infty} a_n (z-z_0)^n$ is the Taylor series expansion of $f(z)$ analytic throughout a disk $|z-z_0|<R_0$, then the coefficients $a_0 =$ _____.

A) 0 B) $f(z_0)$

C) $f'(z_0)$ D) $\frac{f''(z_0)}{2}$

48) If $f(z)=\frac{z^2-6}{(z+1)^6(z^2-1)^5}$ then $z=-1$ is a pole of order _____.

A) 1 B) 6

C) 5 D) 11

49) The value of the integral $\int_C \frac{dz}{z^2(z+3)}$ taken counterclockwise around the circle $|z|=2$ is _____.

A) $\frac{2\pi i}{9}$ B) $2\pi i$

C) $-\frac{2\pi i}{9}$ D) $\frac{2\pi i}{3}$

50) The function $f(z)=\frac{z^3(z^2+1)}{(z+5)(z-4)}$ has _____ simple zeros.

A) $z=0,1,-1$ B) $z=0,i,-i$

C) $z=-5,4$ D) $z=i,-i$

51) If W is a subspace of V then L(W) = _____.

A) W B) V

C) V/W D) W/V

52) The norm of vector $u=(1,-2,5)$ with respect to Euclidean inner product is _____.

A) $\sqrt{30}$ B) $\sqrt{22}$

C) $2\sqrt{5}$ D) $6\sqrt{5}$

53) If V is an inner product space and $u,v\in$ V, then the inequality $|(u,v)|\leq\|u\|\,\|v\|$ is called _____.

A) Triangle inequality B) Cauchy Schwarz inequality

C) Minkowski inequality D) Parallelogram law

54) If T is a linear operator on vector space V. If $\exists\, 0\neq v\in$ V such that $T(v)=cv$, for some $c\in$ F, then *c* is called _____.

A) an Eigen vector of T B) an Eigen value of T

C) rank of T D) none of these

55) If $A=\begin{bmatrix}0 & -1\\ 1 & 0\end{bmatrix}$, then characteristic polynomial of A is _____.

A) x^2 B) x^2+1

C) x^2-1 D) $(x-1)^2$

56) A linear transformation $T:V\to W$ is non singular if _____.

A) T is not one-one B) T is not onto

C) Ker T = {0} D) none of these

57) If $T:V\to W$ and $S:W\to U$ be two linear transformations such that ST is one-one then _____.

A) T is onto B) S is one-one

C) T is one-one D) S is onto

58) If $T : V \to W$ is a linear transformation and if dim Range T = 2 and dim Ker T = 3 then dim V = _____.

A) 6 B) 5

C) 3 D) 2

59) If T is a linear transformation on V_2 defined by $T(x_1, x_2) = (2x_1 + 3x_2, x_1 - x_2)$ then rank of T = _____.

A) 3 B) 0

C) 2 D) 1

60) The vectors (1, 0, 0), (0, 1, 0), (0, 0, 1) are _____.

A) not orthogonal B) linearly independent

C) linearly dependent D) none of these

61) Let G = (V, E) be a graph, where $V = \{a, b, c, d, e\}$ and $E = \{(a,b),(a,e),(b,d),(b,e),(c,d),(d,c),(d,d)\}$ then the degree of vertex *d* is

A) 3 B) 4

C) 5 D) 2

62) In an Euler graph the degree of every vertex is

A) same B) even

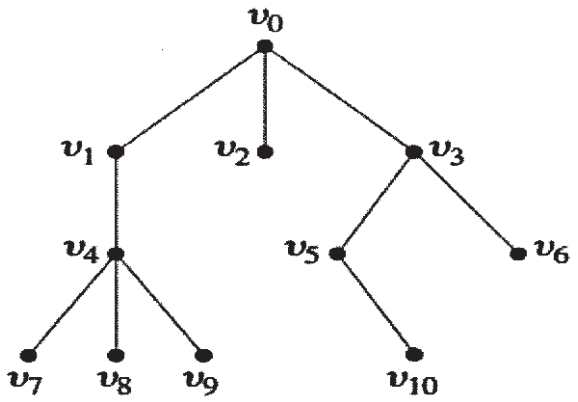
C) odd D) prime

63) If G is any graph with *n* vertices and *m* edges, where *m* and *n* are positive integers and $m \geq n$ then G has

A) a circuit B) component

C) Euler trail D) Hamiltonian Circuit

64) Consider the tree with root v_0 shown below



The number of leaves on the tree are

A) 5 B) 6

C) 4 D) 9

65) What is the maximum number of children that a binary tree node can have?

A) 0 B) 1

C) 2 D) 3

66) Suppose x is a real number. Let $p: 0 < x$, $q: x < 3$, and $r: x = 3$. The the inequality $0 < x \leq 3$ can be written symbolically as:

A) $p \wedge (q \vee r)$ B) $p \wedge r$

C) $p \vee q \vee r$ D) $p \wedge q \wedge r$

67) A compound statement using "AND" is called

A) induction B) disjunction

C) conjunction D) negation

68) Subtraction of binary numbers 11000 and 1011 is

A) 1011 B) 1001

C) 1101 D) 1010

69) The hexadecimal number B09F is equivalent to a binary number

A) 1011000010011111 B) 1011000010011100

C) 1011000010010011 D) 1011000010011001

70) Let p and q are the propositions

p : It is below freezing.

q : It is snowing.

Then the proposition: "It is either below freezing or it is snowing but, it is not snowing if it is below freezing" using p and q and logical connectives can be expressed as

A) $(p \vee q) \wedge (\neg q \rightarrow p)$ B) $(p \vee q) \vee (p \rightarrow \neg q)$

C) $(p \vee q) \vee (\neg q \rightarrow p)$ D) $(p \vee q) \wedge (p \rightarrow \neg q)$

71) The series $\sum_{n=1}^{\infty} \frac{1}{n^2+1}$ is _____.

A) divergent B) convergent

C) oscillatory D) none of these

72) $\int_0^1 x^{m-1}(1-x)^{n-1}\,dx$ is convergent if _____.

A) $m > 0, n < 0$ B) $m < 0, n > 0$

C) $m > 0, n > 0$ D) $m < 0, n < 0$

73) If f is integrable on $[a, b]$ then _____.

A) $\left|\int_a^b f\right| = \int_a^b |f|$ B) $\left|\int_a^b f\right| \geq \int_a^b |f|$

C) $\left|\int_a^b f\right| \leq \int_a^b |f|$ D) $\left|\int_a^b f\right| = -\int_a^b |f|$

74) Characteristic function of all rational on [0, 1] is _____.

A) Riemann integrable
B) not Riemann integrable
C) discontinuous
D) removable discontinuous

75) Generalization of Riemann integral is also called _____.

A) Riemann Stieltjes integral
B) Gamma integral
C) Improper integral of first kind
D) Improper integral of third kind

76) If g is continuous function on $[a, b]$ that is differentiable on (a, b) and if g' is integrable on $[a, b]$ then $\int_a^b g' =$ _____.

A) $g(x)$
B) $g'(b) - g'(a)$
C) $g(a) - g(b)$
D) $g(b) - g(a)$

77) $\lim_{x \to 0} \frac{1}{x} \int_0^x e^{t^2}\, dt =$ _____.

A) e^{x^2}
B) 0
C) 1
D) $2e^{x^2}$

78) If f is integrable on $[a, b]$, then _____.

A) $|f|$ is integrable but converse is not true
B) $|f|$ is not integrable
C) $|f|$ is integrable and converse is true
D) None of these

79) For the function $f(x) = x$ in$[0, 3]$, let P = {0, 1, 2, 3} be the partition of [0, 3]. Then the value of $\mathrm{L}(f, \mathrm{P})$ is _____.

A) 0
B) 3
C) 6
D) 9

80) $\int_b^{\infty} \frac{x^{3/2}}{\sqrt{\left(x^4 - a^4\right)}}\, dx$, where $b > a$, is _____.

A) Convergent
B) Divergent
C) Oscillatory
D) Proper integral

81) The value of $\frac{\left(1+i\sqrt{3}\right)^6}{\left(1-i\sqrt{3}\right)^6}$ is _____.

A) $e^{-\pi i}$
B) 1
C) 1/2
D) 0

82) $\tanh\left(\log\sqrt{2}\right) =$ _____.

A) 0
B) –1
C) 1/2
D) 1/3

83) If $y = a^{5x}$, then the 30th derivative of y is y_{30} = _____.

A) $5^{30} a^{5x}$
B) $5^{30} \left(\log a\right)^{30}$
C) $5^{30} \left(\log a\right) a^{5x}$
D) $5^{30} \left(\log a\right)^{30} a^{5x}$

84) The function $u = \sin\left(\frac{x^2 + y^2}{x + y}\right)$ is _____.

A) a homogeneous function of degree 1
B) a homogeneous function of degree 2
C) a homogeneous function of degree 0
D) not a homogeneous function

85) If a function f is continuous on a closed interval $[a, b]$ and if $f(a)$ and $f(b)$ are of opposite signs then there exists $c \in [a,b]$ such that _____.

A) $f(c) = 0$
B) $f'(c) = 0$
C) $f(a) = f(b)$
D) $f(c) = f(a) = f(b)$

86) If $\frac{1}{\mathrm{N}}\left(\frac{\partial \mathrm{M}}{\partial y}-\frac{\partial \mathrm{N}}{\partial x}\right)$ is function of x alone say $f(x)$, then integrating factor is _____.

A) $f(x)$ B) $e^{\int f(x)\,dx}$

C) $f(y)$ D) $e^{\int f(y)\,dy}$

87) The general solution of $\frac{dy}{dx}+\frac{y}{x}=\frac{1}{x}$ is _____.

A) $xy=x^2+c$ B) $y=1+\frac{c}{x}$

C) $xy=1+c$ D) $y=x+c$

88) The complete solution of $\left(\mathrm{D}^2-9\mathrm{D}+18\right)y=0$ is _____.

A) $y=\mathrm{C}_1e^{3x}+\mathrm{C}_2e^{6x}$ B) $y=\mathrm{C}_1e^{-3x}+\mathrm{C}_2e^{6x}$

C) $y=\mathrm{C}_1e^{3x}+\mathrm{C}_2e^{-6x}$ D) $y=\mathrm{C}_1e^{-3x}+\mathrm{C}_2e^{-6x}$

89) $\frac{1}{\mathrm{D}-a}f(x)=$ _____.

A) $e^{-ax}\int f(x)e^{-ax}\,dx$ B) $e^{-ax}\int f(x)e^{ax}\,dx$

C) $e^{ax}\int f(x)e^{ax}\,dx$ D) $e^{ax}\int f(x)e^{-ax}\,dx$

90) The complete integral of $z=px+qy+\log(pq)$ is _____.

A) $z=ax+by+\log(ab)$ B) $z=ax+y\log a+c$

C) $z=ax-by+\log(ab)$ D) $z=ax-y\log a+c$

91) If $f : \mathrm{R} \to \mathrm{R}$ is given by $(x) = |x|$, then $f^{-1}(2)$ is

A) {4, –4} B) {2, –2}

C) ϕ D) 0

92) If $f(x) = 1 + \cos x (-\infty < x < \infty)$ and $g(x) = x^2 (0 \le x < \infty)$ then $g \circ f(x)$ = _____.

A) $1 + \cos^2 x$ B) $1 + \cos^2 x + 2\cos x$

C) $1 + \cos(x^2)$ D) $1 + \cos(x^2) + 2\cos x$

93) Consider two statements:

(I) Every absolutely convergent series is convergent

(II) Every Cauchy sequence of real numbers is not bounded

A) Only (I) is true B) Only (II) is true

C) Both (I) and (II) are true D) Both (I) and (II) are false

94) If $\{S_n\} = \{\sin(n\pi / 3)\}_{n=1}^{\infty}$, then $\lim_{n\to\infty} \mathrm{Inf}\ S_n$ = _____.

A) $\sqrt{3}/2$ B) $\sqrt{3/2}$

C) $-\sqrt{3/2}$ D) $-\sqrt{3}/2$

95) The series $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n^p}$ converges for _____.

A) P < 0 B) P > 0

C) P = 0 D) P = –1

96) If H is a subgroup of a finite group G & O(H) = 3, O(G) = 30 then [G : H] = ___.

A) 3 B) 6

C) 10 D) 9

97) The order of symmetric group s_5 is _____.

A) 40 B) 60

C) 120 D) 100

98) If $G = \{\pm 1, \pm i\}$ is a multiplicative group then order of –1 is _____.

A) 1 B) 0

C) 2 D) 4

99) The sum of eigen values of a matrix A is _____.

A) equal to trace of A B) equal to the determinant of A

C) equal to rank of A D) 0

100) If A is Skew-Hermitian matrix, then iA is _____.

A) Hermitian B) Skew-Hermitian

C) Symmetric D) Skew-Symmetric

✈ ✈ ✈

Rough Work