Seat No. **ENT-25**

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Shivaji University, Kolhapur M.Sc. Entrance Examination, May-2023 Mathematics (SET-2)

Subject Code: 58716

Day and Date : Tuesday, 08-08-2023

Total Marks : 100

Time : 01.00 p.m. To 02.30 p.m.

Instruction : 1) All questions are compulsory.

- 2) Each question carries 1 marks.
- 3) Choose the correct alternative.
- Let G be a finite group. If H = {x ∈ G | xg = gx for all g ∈ G}, then which of the following statement is not correct?
 (A) H is subgroup of G.
 (B) H is called centre of G
 (C) o(H) divides o(G).
 - (D) H is not abelian.
- 2. If H and K are finite subgroups of a group G, then o(HK) =_____.

$(A) \frac{o(H) + o(K)}{o(H \cap K)}$
$(\Pi) o(H \cap K)$
$(B)\frac{o(H)o(K)}{o(H\cap K)}$
$(D) = o(H \cap K)$
$(\mathbf{C})\frac{o(H\cap K)}{o(H)o(K)}$
$(C) \frac{1}{o(H)o(K)}$
$\rho(H \cap K)$
(D) $\frac{o(H+M)}{o(H)+o(K)}$

3. Let G be a group. Consider the following statements:
I) G is abelian.
II) G' = {e}. Then
(A) Only I) ⇒ II)

(B) Neither I) \Rightarrow II) nor II) \Rightarrow I) (C) I) \Leftrightarrow II)

- $(D) Only II) \Longrightarrow I)$
- 4. $o(S_3) =$ _____. (A) 3 (B) 6 (C) 2
 - (D) ∞

5. $Z_p = \{0, 1, 2, \dots, p-1\} \text{ modulo } p \text{ is a field if } p \text{ is } ___.$

- (A) finite
- (B) odd
- (C) even
- (D) prime

- (1) -

- 6. An element e in a ring R is called _____ if e² = e.
 (A) unit
 (B) invertible
 (C) idempotent
 - (D) nilpotent
- 7. A ring R ≠ {0} is called a _____ ring if R has no ideals except R and {0}
 (A) boolean
 (B) commutative
 (C) division
 - (D) simple
- 8. Let f: R → R' be a ring homomorphism. Consider the following statements:
 I) Ker f is an ideal of R.
 II) Ker f = {0} ⇒ f is one one. Then
 (A) Both I) and II) are true.
 (B) Both I) and II) are false.
 (C) Only I) is true.
 (D) Only II) is true.
- 9. Consider the following statements:

 1) 8x³ + 6x + 1 ∈ Z[x] is primitive.
 II) 8x³ + 6x + 2 ∈ Z[x] is primitive. Then
 (A) Both I) and II) are true.
 (B) Both I) and II) are false.
 (C) Only I) is true.
 (D) Only II) is true.

10. Let $f(x) = x + 1 \in Z_2[x]$. Then $[f(x)]^2 = _$ in $Z_2[x]$. (A) 1 (B) $1 + x^2$ (C) x^2 (D) $1 - x^2$

11. If z = -1 then its Principal argument is Arg (z) = ...
A) π B) π/2 C) 2π D) 0
12. If (z) = u(x, y) + iv(x, y), then the Cauchy-Riemann equations are ...
A) u_x = -v_y, u_y = v_x B) u_x = v_y, u_y = v_x

C)
$$u_x = -v_y$$
, $u_y = -v_x$ D) $u_x = v_y$, $u_y = -v_x$

13. If
$$f(z) = x^3 + i(1 - y)^3$$
, then $f'(i) = \cdots$
A) i B) $-i$ C) 0 D) \bar{z}

14. If f(z) = u(x, y) + iv(x, y) is an analytic function such that $\overline{f}(z) = u(x, y) - iv(x, y)$ is also analytic then the function f(z) is... D) an identity function A) does not exist B) constant C) only a zero function 15. The value of the contour integral $\int_C \frac{dz}{z} = \cdots$, where C is the top half $z = e^{i\theta} (0 \le \theta \le \pi)$ of the circle |z| = 1 from z = 1 to z = -1. C) $-\pi i$ B) $2\pi i$ A) πi D) $-2\pi i$ 16. If C is the arc of the circle |z| = 2 from z = 2 to z = 2i lying in the first quadrant then $\left| \int_C \frac{dz}{z^{4}+1} \right| \leq \dots$ B) $\frac{2}{15}\pi$ D) $\frac{4}{15}\pi$ C) $\frac{3}{15}\pi$ A) $\frac{1}{15}\pi$ 17. The series of complex numbers $\sum_{n=0}^{\infty} \left(\frac{1}{2} + i\frac{1}{2}\right)^n$ is... B) converges to $\frac{1}{\frac{1}{1+i^{\frac{1}{2}}}}$ A) divergent C) converges to $\frac{1}{\frac{1}{1-i\frac{1}{1-i}}}$ D) converges to $\frac{-1}{\frac{1}{2}+i\frac{1}{2}}$ 18. If $f(z) = e^{1/z}$ then z = 0 is ... singularity of f(z)A) Removable B) simple Pole C) Pole of any finite nonzero order D) essential 19. The value of the integral $\int_C \frac{(z+2) dz}{z^2(z+3)}$ taken counter clockwise around the circle |z| = 2is A) $\frac{2\pi i}{q}$ D) $\frac{2\pi i}{3}$ B) 2πi C) $-\frac{2\pi i}{2\pi i}$ 20. The function $f(z) = \frac{z^3(z^2+1)}{(z+5)(z-4)}$ has simple zeros. C) z = -5, 4 D) z = i, -iA) z = 0, 1, -1 B) z = 0, i, -i21. If L{ f(t) } = f(s) then L{ f(t) / t} = A) $\int_{s}^{\infty} f(t) ds$ B) $\int_{s}^{\infty} f(s) dt$ C) $\int_{s}^{\infty} f(s) ds$ D) $\int_{c}^{\infty} f(st) dt$ 22. $L\{y''(t)\} = \dots$ A) $s^2 L\{y\} - s y(0) - y'(0)$ B) $s L\{y\} + y(0)$ C) $sL\{y\} - y(0)$ D) $s^2 L\{y\} + sy(0) + y'(0)$

23. L{
$$1/\sqrt{\pi t}$$
 } =
A) $\frac{2}{\sqrt{s}}$
B) $\frac{1}{\sqrt{2s}}$
D) $\frac{1}{s}$
24. L^{-1} { $\frac{1}{\sqrt{2s+1}}$ } =

$$\begin{array}{ccc}
 L & \{ \frac{1}{s^2 (s^2 + 1)} \} = & \dots \\
 A) & t - \cos t & B) & t - \sin t \\
 C) & t - 2\sin t & D) & t + \cos t
\end{array}$$

C) t - 2sint DJ t + cost25. Infinite Fourier transform of <math>F(x) = 1, |x| < k

$$= 0$$
 , $|x| > k$

where
$$F{F(x)} = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} F(x) e^{isx} dx$$

A) $\sqrt{\frac{2}{\pi}} \frac{cossk}{s}$
B) $\sqrt{\frac{2}{\pi}} \frac{tansk}{s}$
C) $\sqrt{\frac{2}{\pi}} \frac{sinsk}{s}$
D) $\sqrt{\frac{2}{\pi}} \frac{sinsk}{k}$

26. The circular $\sin\theta = \dots$

A)
$$\frac{e^{i\theta} - e^{-i\theta}}{2i}$$

B) $\frac{e^{i\theta} - e^{-i\theta}}{2}$
C) $\frac{e^{i\theta} + e^{-i\theta}}{2i}$
D) $\frac{e^{i\theta} + e^{-i\theta}}{2}$

27. If L { f(t) } = f(s) then L { f(at) } = $\frac{1}{a}$ f(s/a). This is A) change of scale property C) effect of division B) second shifting theorem D) first shifting theorem

28. L { $e^{at}t^n$ } = s > a

A)
$$\frac{n!}{(s-a)^{n+1}}$$

B) $\frac{n!}{(s+a)^{n+1}}$
C) $\frac{n}{(s-a)^{n+1}}$
D) $\frac{n!}{(s-a)^n}$

29. The value of $\int_0^\infty te^{-3t} sint dt = \dots$

A)	$\frac{11}{50}$	в)	$\frac{10}{49}$
C)	$\frac{3}{50}$	D)	<u>12</u> 35

30. Infinite Fourier sine transform of $F(x) = \frac{1}{x}$ over $0 < x < \infty$ is

where $f_s(s) =$	$\sqrt{\frac{2}{\pi}} \int_0^\infty F(x) \sin x dx$	
A) $\sqrt{\frac{\pi}{4}}$	B) ,	\int_{π}^{2}
C) $\sqrt{\frac{\pi}{2}}$	D)	$\sqrt{\frac{3}{\pi}}$

31. If W is a subspace of V then there exists an onto linear transformation θ : V $\rightarrow \frac{V}{W}$ such that ker θ = _____.

A) V B)
$$\frac{V}{W}$$
 C) $\frac{W}{V}$ D) W

32. If $T: V \rightarrow U$ is a homomorphism, then ker $T = \{0\}$ iffA) T is one - oneB) T is ontoC) T is neither one - one nor ontoD) none of these

33. The norm of vector (4, 2, 2, -6) with respect to Euclidean inner product is _____.A) 60B) $2\sqrt{15}$ C) 14D) $4\sqrt{15}$

34. If V is an inner product space and $x, y \in V$ are orthogonal vectors, then $||x + y||^2 =$ _____

A) $2(x ^2 + y ^2)$	B) $ x ^2 - y ^2$
C) $2(x ^2 - y ^2)$	D) $ x ^2 + y ^2$

35. Let $B = \{e_1 = (1, 0, 0), e_2 = (0, 1, 0), e_3 = (0, 0, 1)\}$ be the standard ordered basis for \mathbb{R}^3 and $F: \mathbb{R}^3 \to \mathbb{R}^3$ be a linear transformation. If $F(e_1) = (1, 3, 5), F(e_2) = (2, 4, 6), F(e_3) = (7, 7, 7)$, then the matrix of F is $[F]_B = \dots$

A)	[1 2 7	3 4 7	5 6 7	C)	[1 3 5	2 4 6	7 7 7
B)	2 7 1	4 7 3	6 7 5	D)	$\begin{bmatrix} 1\\ 0\\ 0 \end{bmatrix}$	0 1 0	0 0 1

36. Let c be an eigen value of a linear operator T on V. Then the set $\{v \in V \mid T(v) = cv\}$ is called of T.

A) eigen space	C) range
B) null space	D) kernel

37. If T: V \rightarrow W is a line Rank T =	inear transformation a	and if dim $V = 5$ and	Nullity $T = 3$, then	
A) 8	B) 2	C) 3	D) 5	
38. If V is an Inner product	space and $x, y \in V$ the	n $ x+y ^2 + x-y ^2 = $	·	
A) $2(x ^2 - y ^2)$	B) $ x ^2 + y ^2$	C) $2(x ^2 + y ^2)$	D) $ x ^2 - y ^2$	
39. If T : V \rightarrow U is a linear	transformation, then R	ange T is	·	
A) a subspace of U	B) equal to V	C) a subspace of V	D) none of these	
40. If $T: V_3 \rightarrow V_2$ and $S:$	$V_3 \rightarrow V_2$ are two li	near transformations def	ined by	
$T(x_1, x_2, x_3) = (x_1) (S+T)(x_1, x_2, x_3) =$	$(x_1 - x_2, x_2 + x_3)$ and	$S(x_1, x_2, x_3) = (2x_1)$	$_{1}$, $x_{2} - x_{3}$) then	
		C) (x_1, x_2, x_3) I	D) $(3x_1 - x_2, 2x_2)$	
41. The maximum degree of	any vertex in a simple	graph with n vertices is		
A) n-1	B) n+1	C) 2n-1	D) n	
42. Let G be a simple graph.P: Adjacency matrix is sQ: Trace of adjacency m	ymmetric.	g statements is true?		
A) P only	B) Q only	C) Both P and Q	D) Neither P nor Q	
43. A full binary tree of heig	tht h has leaves			
A) 2h	B) h-1	C) h	D) 2 ^h	
44. Let the graphs G_1 and G_2	be isomorphic. Then			
A) both G_1 and G_2 have s	same number of vertice	es and edges.		
B) both G_1 and G_2 have same number of circuits.				
C) both G ₁ and G ₂ have same number of odd vertices.				
D) all the options are cor	rect.			
45. How many non-isomorph of each vertex is 2?		-	es and the degree	
A) 2 B) 3	C) 4	D) 5		
46. The binary number 1001 A) 3DB9 B) 4I				
47. Which of the following is not the rule of inference?A) Modus Ponens B) Transitivity C) Elimination D) Contradiction				

Which of the following statements is a Tautology?

(A) $(p \to q) \leftrightarrow (\neg q \to \neg p)$ (B) $(p \to q) \to (p \land q)$ (C) $p \land \neg q$ (D) $(p \land \neg q) \land (\neg p \lor q)$

49. Let P(x) denote the statement x > 7: Which of these have truth value true?A) P(0) B) P(4) C) P(6) D) P(9)

50. What is the nature of a logical argument? Choose the correct answer from the following.A) Justified or unjustified B) True or false C) Valid or invalid D) Verifiable or not verifiable

51. If $f(x) = x^2$ for all $x \in [0, 1]$ and $D = \{0, \frac{1}{4}, \frac{2}{4}, \frac{3}{4}, 1\}$ be a partition of [0, 1], then L(f, D) and U(f, D) are respectively

A)
$$\frac{15}{32}$$
, $\frac{7}{32}$ B) $\frac{7}{32}$, $\frac{15}{32}$ C) 15, 7D) 7, 15

52. If g is continuous function on [a, b] that is differentiable on (a, b) and if g' is integrable on [a, b] then $\int_a^b g' = \dots$

- A) g(x)B) g'(b) - g'(a)C) g(a) - g(b)D) g(b) - g(a)
- 53. Every constant function f(x) = c is on any interval [a, b].
- A) Riemann integrable B) not Riemann integrable
- C) discontinuous D) removable discontinuous

54. Let f be a bounded function defined on [a, b] and let P and P^* be the partitions of [a, b]. If $P \subseteq P^*$, then

A) $L(f, P^*) \le L(f, P)$ B) $L(f, P^*) = L(f, P)$ C) $U(f, P^*) \le U(f, P)$ D) $U(f, P^*) \ge U(f, P)$

48.

- 55. $\int_{b}^{\infty} \frac{x^{3/2}}{\sqrt{(x^{4}-a^{4})}} dx$, where b > a, is A) Convergent B) divergent C) Oscillatory D) proper integral
- 56. The series $\sum_{n=1}^{\infty} \frac{1}{n^2+1}$ isB) convergentA) divergentB) convergentC) oscillatoryD) none of these
- 57. The mesh of partition P is of the subinterval comprising P.A) Minimum length B) Maximum length C) Equal D) none

58. When f(x) is odd function then for Fourier series in $[-\pi, \pi]$, then $a_0 = \dots$ A) 0 B) 2 C) ∞ D) -1

59. If f(x) is expanded in a Fourier series of

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx$$

Then $a_0 =$

- C) $\frac{2}{\pi} \int_0^{\pi} f(x) dx$ D) $\frac{\pi}{2} \int_0^{\pi} f(x) dx$
- 60. $\int_0^1 x^{m-1} (1-x)^{n-1} dx$ is convergent if

A) m > 0, n < 0B) m < 0, n > 0C) m > 0, n > 0D) m < 0, n < 0

- 61. A sequencing involving six jobs and three machines requires evaluation of :
 - A) (6!+6!+6!) sequences
 - B) $(6!)^3$ sequences
 - C) $(6)^3$ sequences
 - D) (6+6+6) sequences
 - 62. In sequencing algorithm_
 - A) The selection of an appropriate order for a series of jobs is to be done on a finite service facilities
 - B) All the jobs must be processed on a first-come-first service basis
 - C) A service facility can process more than one job at a time
 - D) All the service facilities are not of different type

- 63. In sequencing problems _____.
 - A) All jobs are completely known and ready for processing.
 - B) Jobs are processed sequentially, i.e., first on the first machine and then on the second machine and so on.
 - C) Total elapsed time is determined by the point of time at which the first of the n jobs goes to machine A until the time when the last job comes of the machine B.
 - D) All of the above.

64. Which statement is true about the game $\begin{bmatrix} 5 & 0 \\ 0 & 2 \end{bmatrix}$?

- A) Game is fair.
- B) Game is strictly determinable.
- C) Game is not strictly determinable.
- D) All of the above.

65. Using Dominance method following matrix can be reduced to

[10	5	-2]
13	12	15
L16	14	10

- A) $\begin{bmatrix} 10 & 5 \\ 13 & 12 \end{bmatrix}$ B) $\begin{bmatrix} 5 & -2 \\ 12 & 15 \end{bmatrix}$ C) $\begin{bmatrix} 13 & 15 \\ 16 & 10 \end{bmatrix}$ D) $\begin{bmatrix} 12 & 15 \\ 14 & 10 \end{bmatrix}$

66. To convert unbalanced transportation problem with total supply equals to 400 & total demand equals to 500 into balanced problem we add .

A) dummy column with demand 100

B) dummy column with demand 200

- C) dummy row with supply 100
- D) dummy row with supply 200
- 67. Every basic feasible solution of a general assignment problem, having a square payoff matrix of order n should have assignments equal to _____.
 - A) 2n+1
 - B) m+n
 - C) m+n-1
 - D) 2n-1

68. For a salesman, who has to visit n cities, following are the ways of his tour plan:

B) n! C) (n-1)! A. n D) (n+1)! 69. An optimization model .

- A§ Mathematically provides the best decision
- B) Provides decision within its limited context
- C) Helps in evaluating various alternatives constantly
- D) All of the above

70. A constraint in an LPP restricts

- A) Value of an objective function
- B) Value of a decision variable
- C) Use of available resources
- D). Uncertainty of optimum value

71. The set \overline{E} of all limit points of E is called the _____

A) open set B) Closure of E C) connected set

D) compact

72. The union of countable collection of closed sets is

A) always open B) always closed

C) need not be closed D) neither closed nor open

73. If f and g are continuous functions from a metric space M_1 into a metric space

M₂ then which of the following statement is false?

A) f + g is always continuous on M_1

B) f - g is always continuous on M_1

C) f.g is always continuous on M_1

D) $\frac{f}{a}$ is always continuous on M₁

74. The set in metric space X is open if and only if its complement is ______

A) closed B) open C) always empty set D) always X

75. Every subset of R_d is -----.

A) both open and closed B) only open

C) only closed D) neither open nor closed.

76. Let $\langle X, d \rangle$ and $\langle Y, \rho \rangle$ be two metric spaces and $f: X \to Y$ is function. Then f is continuous if and only if $f^{1}(G)$ is open in X whenever ------.

A) G is closed in X	B) G is open in X
C) G is neither open nor closed	D) G is open in Y

77. If f is continuous at a and if $c \in \mathbb{R}$ then cf is continuous A) on R C) at c B) at a D) at a.c 78. In any metric space $\langle M, \rho \rangle$ A) Both M and empty set are open sets. B) Only M is open set C) Only Empty set is open set. D) Both are neither open nor closed sets. 79. If E is any subset of metric space M then which of the following statement is true ? A) E $\subset \overline{E}$ B) E is closed subset of M if $E = \overline{E}$ C) E is closed and $\overline{E} = \overline{\overline{E}}$ D) All the statements in A), B) and C) are true. 80. Let $\langle X, d \rangle$ and $\langle Y, \rho \rangle$ be two metric spaces and $f: X \rightarrow Y$ be a function. Then f is continuous if and only if $\overline{f^{-1}(B)} \subset f^{-1}(\overline{B})$ for every ------. A) set B of X B) subset B of X C) set B of Y D) subset B of Y 81. If $f: A \rightarrow B$ and if $X \subseteq B, Y \subseteq B$ then A) $f^{-1}(X \cap Y) \neq f^{-1}(X) \cap f^{-1}(Y)$ B) $f^{-1}(X \cup Y) = f^{-1}(X) \cap f^{-1}(Y)$ C) $f^{-1}(X \cap Y) = f^{-1}(X) \cup f^{-1}(Y)$ D) $f^{-1}(X \cup Y) = f^{-1}(X) \cup f^{-1}(Y)$ 82. If $f(x) = 1 + \cos x$ ($-\infty < x < \infty$) and $g(x) = x^2 (0 \le x < \infty)$ then $g \circ f(x) = \dots$ $c) = 1 + cos(x^2)$ A) $1 + cos^2 x$ B) $1 + \cos^2 x + 2 \cos x$ D) $(1 + \cos(x^2) + 2 \cos x)$ 83. If $\{S_n\} = \{n (1 + (-1)^n)\}_{n=1}^{\infty}$, then $\lim_{n \to \infty} \ln f S_n =$ _____ A) ∞ B] -1 C[0 **D**1 84. Consider two statements I) Every absolutely convergent series is convergent II) Every Cauchy sequence of real numbers is not bounded A) Only I) is true B) Only II) is true C) Both I) and II) are true (D) Both I) and II) are false 85. The series $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n^p}$ converges for _____ A P < 0 $\overrightarrow{B}P > 0$ $(\mathbf{\dot{C}}) P = 0$ D) P = -186. The factor group of an abelian group is A)cyclic B neither abelian nor cyclic D) none of these C) abelian

- (11) -

- 87. A non-zero element 'a' of a commutative ring R is called a if there exists some non zero element b in R such that ab=0. d nilpotent A) Zero divisor B) no zero divisor D) idempotent 88. The order of symmetric group s_5 is.... A) 40 B) 60 C)120 D)100 89. The diagonal elements of the matrix are either purely imaginary or zero. A) Hermitian B) Skew- Hermitian C) symmetric D) Skew- symmetric 90. For the Euler's function ϕ , $\phi(16) =$ ______ A\ 6 B\ 7 C\ 8 D) 9 91. The integrating factor of the differential equation $\frac{dy}{dx} + \frac{2x}{1+x^2} y = \sin x$ is ______.
 - A) $1 + x^2$ B) $\frac{1}{1 + x^2}$ C) $\frac{2x}{1 + x^2}$ D) $\frac{1 + x^2}{2x}$

92. Which of the following is Bernoulli's differential equation?

A) $\frac{dy}{dx} + Py = Q$ B) $\frac{dy}{dx} + Px = Q$ C) $\frac{d^2y}{dx^2} + Py = Q$ D) $\frac{dy}{dx} + Py = Qy^n$

93. The Particular integral of $(D - 2) (D + 2) y = e^{2x}$ is _____.

A) e^{2x} B) $\frac{1}{4}e^{2x}$ C) $\frac{x}{4}e^{2x}$ D) xe^{2x}

94. The differential equation $x^2 \frac{d^2 y}{dx^2} - 4x \frac{dy}{dx} + 6y = x$ is of the type _____

A) Legendre's linear differential equation

B) Homogeneous differential equation.

C) Linear differential equation

D) Bernoulli's differential equation

- (12) -

95. The complete solution of p + q + pq = 0 is _____.

A) z = ax + ay + cB) $z = ax + \frac{a}{a+1}y + c$

C) z = ax + c

B)
$$z = ax + \frac{a}{a+1}y + c$$

D) $z = ax - \frac{a}{a+1}y + c$

96. A continuous function defined on a closed interval is _____.

- A) uniformly continuous
- B) unbounded
- C) bounded
- D) All of these
- 97. $\lim_{x \to 1} \frac{\log x}{x-1} = \underline{\qquad}$ A) ∞ B) 1
 C) 0
 D) 2

98. The functions $f(x) = x^2$ and g(x) = x satisfies all conditions of Cauchy's mean value theorem on [a, b]. Then the value of c =_____.

A)
$$\frac{a-b}{2}$$

B) $\frac{a+b}{2}$
C) ab
D) $\frac{b-a}{2}$

99. If $y = a^{5x}$, then the 30th derivative of y is $y_{30} =$ _____.

- A) $5^{30}a^{5x}$
- B) $5^{30}(\log a)^{30}$
- C) $5^{30}(\log a)a^{5x}$
- D) $5^{30}(\log a)^{30}a^{5x}$

Ζ

100. If $z = \frac{xy}{x+y}$ then $x \frac{\partial z}{\partial x} + y \frac{\partial z}{\partial y} =$ _____.

- A)
- B) 0
- C) 2*z*
- D) z^2