

Seat No.	
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P.G. Entrance Examination, May - 2023

M.Sc. MATHEMATICS

Sub. Code : 58716

Day and Date : Monday, 08-05-2023

Total Marks : 100

Time : 1.00 p.m. to 2.30 p.m.

- Instructions :**
- 1) All questions are compulsory.
 - 2) Each question carries 1 mark.
 - 3) Answers should be marked in the given OMR answer sheet by darkening the appropriate option.
 - 4) Follow the instructions given on OMR Sheet.
 - 5) Rough work shall be done on the sheet provided at the end of question paper.

- 1) Let G be a group. Consider the following statements :
 - I) G is abelian
 - II) $Z(G) = G$. Then
 - A) Only $(I) \Rightarrow (II)$
 - B) Neither $(I) \Rightarrow (II)$ nor $(II) \Rightarrow (I)$
 - C) $(I) \Leftrightarrow (II)$
 - D) Only $(II) \Rightarrow (I)$

- 2) Consider a group $G = \{1, -1, i, -i\}$ under usual multiplication. Which of the following statement is not correct?
 - A) G is a cyclic group
 - B) $G = \langle i \rangle$
 - C) $G = \langle -i \rangle$
 - D) $G = \langle 1 \rangle$

- 3) Let $G = S_3$. Which of the following statement is not correct?
 - A) $G' = A_3$ is a cyclic group
 - B) G is abelian
 - C) A_3 is normal subgroup of G
 - D) $\frac{G}{A_3}$ is abelian

- 4) Consider the following statements :
- I) Field is an integral domain.
 II) Integral domain is a field. Then
- A) Both (I) and (II) are true B) Both (I) and (II) are false
 C) Only (I) is true D) Only (II) is true
- 5) Let R be a ring of integers. Characteristic of R is _____ .
- A) 0 B) p (a prime number)
 C) 1 D) ∞
- 6) An element a in a ring R is called _____ if $a^n = 0$ for some integer n .
- A) unit B) invertible
 C) idempotent D) nilpotent
- 7) Let $f: Z \rightarrow Z$ defined by $f(x) = 0$ for all $x \in Z$, where Z is ring of integers. Consider the following statements :
- I) f is a homomorphism
 II) f is onto. Then
- A) Both (I) and (II) are true B) Both (I) and (II) are false
 C) Only (I) is true D) Only (II) is true
- 8) Let R be a ring. If $f(x), g(x) \in R[x]$ be non-zero polynomials such that $f(x) + g(x) \neq 0$, then which of the following statement is true?
- A) $\deg(f(x) + g(x)) \leq \min[\deg(f(x)), \deg(g(x))]$
 B) $\deg(f(x) + g(x)) \leq \max[\deg(f(x)), \deg(g(x))]$
 C) $\deg(f(x) + g(x)) > \max[\deg(f(x)), \deg(g(x))]$
 D) $\deg(f(x) + g(x)) \leq 0$

34) Infinite inverse Fourier sine transform of e^{-as} over $0 < s < \infty$ is _____ where

$$F(x) = \sqrt{\frac{2}{\pi}} \int_0^{\infty} f_s(s) \cdot \sin sx \, ds$$

A) $\sqrt{\frac{2}{\pi}} \cdot \frac{x}{(a^2 + x^2)}$

B) $\sqrt{\frac{2}{\pi}} \cdot \frac{x}{(a^2 - x^2)}$

C) $\sqrt{\frac{2}{\pi}} \cdot \frac{1}{(a^2 + x^2)}$

D) $\sqrt{\frac{2}{\pi}} \cdot \frac{1}{(a^2 - x^2)}$

35) If $L\{f(t)\} = f(s)$ and $G(t) = f(t - a) \quad t > a$
 $\quad \quad \quad \quad \quad = 0 \quad \quad \quad t < a$

then $L\{G(t)\} =$ _____

A) $e^{as} \cdot f(s)$

B) $e^{-as} \cdot f(s)$

C) $e^{-s} \cdot f(s)$

D) $e^{-as} \cdot f'(s)$

36) $L\{e^{-4t} \cdot t^4\} =$ _____

A) $\frac{4}{(s-4)^4}$

B) $\frac{5!}{(s-3)^4}$

C) $\frac{5!}{(s+3)^4}$

D) $\frac{4!}{(s+4)^5}$

37) $L^{-1}\left\{\frac{1}{(s-3)^2}\right\} =$ _____

A) $e^{2t} \cdot t$

B) $e^{3t} \cdot t$

C) $e^{3t} \cdot t^2$

D) $e^t \cdot t$

38) If $L\{f(t)\} = f(s)$ then $L^{-1}\{f(as)\} =$ _____

- A) $a L^{-1}\left\{f\left(\frac{s}{a}\right)\right\}$ B) $2a L^{-1}\left\{f\left(\frac{s}{a}\right)\right\}$
 C) $\frac{1}{a} L^{-1}\left\{f\left(\frac{s}{a}\right)\right\}$ D) $\frac{1}{a} L^{-1}\{f(s)\}$

39) $L^{-1}\left\{\frac{1}{(s^2 + 9)}\right\} =$ _____

- A) $\frac{\sin 3t}{9}$ B) $\frac{\sin 3t}{3}$
 C) $\frac{\sin 9t}{3}$ D) $\frac{\sin 9t}{9}$

40) $L^{-1}\left\{\log\left(\frac{s+4}{s+3}\right)\right\} =$ _____

- A) $\frac{1}{t}\{e^{-3t} + e^{-2t}\}$ B) $\frac{-2}{t}\{e^{-4t} - e^{-3t}\}$
 C) $\frac{2}{t}\{e^{-3t} + e^{-4t}\}$ D) $\frac{-1}{t}\{e^{-4t} - e^{-3t}\}$

41) In 'n' jobs and two machines (say A and B) sequencing problems in which the order of processing is AB.

- A) Job having minimum time on machine B is processed first
 B) Job having minimum time on machine A is processed in the last
 C) Job having minimum time on machine B is processed in the last
 D) Job having maximum time on machine B is processed in the last

- 69) The series $\sum_{n=1}^{\infty} \frac{1}{n^2 + 1}$ is _____
- A) divergent
B) convergent
C) oscillatory
D) none of these
- 70) $\int_{\pi}^{\infty} \frac{\sin^2 x}{x^2} dx$ is _____
- A) convergent
B) divergent
C) oscillatory
D) proper integral
- 71) Let $\langle X, d \rangle$ and $\langle Y, \rho \rangle$ be two metric spaces and $f : X \rightarrow Y$ be a function. Then f is continuous if and only if $\overline{f^{-1}(B)} \subset f^{-1}(\overline{B})$ for every _____ .
- A) set B of X
B) subset B of X
C) set B of Y
D) subset B of Y
- 72) If E is any subset of metric space M then which of the following statement is true?
- A) $E \subset \overline{E}$
B) E is closed subset of M if $E = \overline{E}$
C) E is closed and $\overline{\overline{E}} = \overline{E}$
D) All the statements in (A), (B) and (C) are true
- 73) Every subset of R_d is _____.
- A) both open and closed
B) only open
C) only closed
D) neither open nor closed
- 74) Let $\langle M_1, \rho_1 \rangle$ and $\langle M_2, \rho_2 \rangle$ be metric spaces and let $f : M_1 \Rightarrow M_2$. Then f is continuous on M_1 if and only if $f^{-1}(G)$ is open in _____, whenever G is open in M_2 .
- A) M_1
B) M_2
C) $f(M_1)$
D) $f(M_2)$

- 75) Any polynomial function is _____ at each point in \mathbb{R}^1 .
 A) continuous B) discontinuous
 C) not differentiable D) always constant
- 76) The set in metric space X is closed if and only if its complement is _____.
 A) closed B) open
 C) always empty set D) always X
- 77) The intersection of countable collection of open sets is
 A) always open B) always closed
 C) need not be open D) neither closed nor open
- 78) The set \bar{E} of all limit points of E is called the _____.
 A) open set B) closure of E
 C) connected set D) compact
- 79) If $a \in \mathbb{R}_d$ then $\{a\}$ is _____.
 A) $B[a;1]$ B) open set in \mathbb{R}_d
 C) not open set in \mathbb{R}_d D) not closed set in \mathbb{R}_d
- 80) If f is continuous at a and if $c \in \mathbb{R}$ then cf is continuous
 A) on \mathbb{R} B) at a
 C) at c D) at $a.c$
- 81) The greatest lower bound of the set $\{1/2^n \mid n \in \mathbb{N}\}$ is _____.
 A) $\frac{1}{2}$ B) 1
 C) 0 D) 2

- 96) The complete integral of $z = px + qy + \log(pq)$ is _____.
- A) $z = ax + by + \log(ab)$ B) $z = ax + y \log a + c$
 C) $z = ax - by + \log(ab)$ D) $z = ax - y \log a + c$
- 97) The order of partial differential equation $\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2 = 3z$ is _____.
- A) 2 B) 4
 C) 1 D) 3
- 98) The complementary function of $x^2 \frac{d^2 y}{dx^2} - 2x \frac{dy}{dx} + 2y = x$ is _____.
- A) $C_1 e^x + C_2 e^{2x}$ B) $C_1 e^{-x} + C_2 e^{-2x}$
 C) $C_1 x + C_2 x^2$ D) $C_1 + C_2 x$
- 99) $\frac{1}{D-a} f(x) =$ _____.
- A) $e^{-ax} \int f(x) e^{-ax} dx$ B) $e^{-ax} \int f(x) e^{ax} dx$
 C) $e^{ax} \int f(x) e^{ax} dx$ D) $e^{ax} \int f(x) e^{-ax} dx$
- 100) If $\frac{1}{N} \left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x} \right)$ is function of x alone say $f(x)$, then integrating factor is _____.
- A) $f(x)$ B) $e^{\int f(x) dx}$
 C) $f(y)$ D) $e^{\int f(y) dy}$



Rough Work