Seat No.	

M.Phil./Ph.D. Entrance Examination, September - 2022
MATHEMATICS
Sub. Code : 58796

Day and Date : Saturday, 24 - 09 - 2022　　　　　　**Total Marks : 100**
Time : 01.00 p.m. to 03.00 p.m.

Instructions :
1) All questions are compulsory.
2) Each question carries 2 marks.
3) Answers should be marked in the given OMR answer sheet by darkening the appropriate option.
4) Use black ball point pen only for marking the circle. Do not make any stray mark on the OMR Answer Sheet.
5) Follow the instructions given on OMR Sheet.
6) Rough work shall be done on the sheet provided at the end of question paper.
7) Only non-programmable calculators are allowed.

1) Video conferencing can be classified as one of the following types of communication.

    (A) Visual one way　　　　　(B) Audio-visual one way

    (C) Audio-visual two way　　　(D) Visual two way

2) The main objective of organizing a workshop is

    (A) To improve the skills of the researchers in order to solve the specific problem

    (B) To impart practical training to the researchers

    (C) To make perfect the researchers in an area of research

    (D) All of the above

P.T.O.

3) Action research is

   (A) An applied research

   (B) A research carried out to solve immediate problems

   (C) A longitudinal research

   (D) Simulative research

4) Field study is related to

   (A) Real life situation   (B) Experimental research

   (C) Conceptual research   (D) Fundamental research

5) Research is a

   (A) Value oriented process

   (B) Passive process

   (C) Self-contained process

   (D) Discovery oriented process

6) Attributes of the objects, events or things which can be measured are called

   (A) quantitative measures

   (B) qualitative measures

   (C) variables

   (D) data

7) The format of research resembles with

   (A) A long essay

   (B) A synopsis

   (C) A writing technique

   (D) A report preparing method

8) The research papers are written in order to

   (A) Gain name and fame

   (B) Communicate the research

   (C) Get promotions

   (D) None of the above

9) The research work is not influenced by

   (A) The researcher himself

   (B) His pre-established assumptions

   (C) The curse of some other researchers

   (D) The wrong calculations

10) The Ethics in research is not related to

   (A) Self-interest

   (B) Scientific method

   (C) Reliability

   (D) Humanity

11) The research reporting should be carried out

   (A) In a scientific way

   (B) In an imaginary way

   (C) Through copying

   (D) Through discussion among the scientists

12) Generally the formulation and progress of new knowledge have been carried out through

(A) Fundamental researches

(B) Experimental researches

(C) Historical researches

(D) None of the above

13) Which of the following is the correct mathematical writing?

(A) $\sin x = x\pi_{n=1}^{\infty}(1-x^2\pi^{-2}n^{-2})$

(B) $\sin x = x\Pi_{n=1}^{\infty}(1-x^2\pi^{-2}n^{-2})$

(C) $\sin x = x\pi_{n=1}^{\infty}(1-x^2\Pi^{-2}n^{-2})$

(D) $\sin x = x\Pi_{n=1}^{\infty}(1-x^2\Pi^{-2}n^{-2})$

14) Which of the following is an example of professional writing for book in bibliography?

(A) J.B. Fraleigh, *A First Course in Abstract Algebra,* Pearson Education, 2003.

(B) J.B. Fraleigh, *A First Course In Abstract Algebra,* Pearson Education, 2003.

(C) J.B. Fraleigh, *A first course in abstract algebra,* Pearson Education, 2003.

(D) J.B. Fraleigh *A First Course in Abstract Algebra,* Pearson Education, 2003.

15) Which of the following is an example of professional writing?

(A) Consequently $R_{ij} = 0$ characterizes the empty space

(B) Consequently, $R_{ij} = 0$ characterizes the empty space

(C) Consequently, $R_{ij} = 0$, characterizes the empty space

(D) Consequently, $R_{ij} = 0$ Characterizes the empty space

16) Correct usage of conjunction "if and only if" is _____

   (A) "if and only if" or "iff"

   (B) "if, and only if"

   (C) "if, and only if,"

   (D) "if, and, only if"

17) Similarity checks for plagiarism shall exclude

   (A) All references, bibliography, table of content, preface and acknowledgements

   (B) All quoted work either falling under public domain or reproduced with all necessary permission and/or attribution

   (C) All generic terms, laws, standard symbols and standards equations

   (D) All of the above

18) Which of the following is an example of professional writing?

   (A) This completes the proof of my Inequalities (5.2) and (5.4)

   (B) This completes the proof of our Inequalities (5.2) and (5.4)

   (C) This completes the proof of our inequalities (5.2) and (5.4)

   (D) This completes the proof of my inequalities (5.2) and (5.4)

19) Text after the command \\ in latex

   (A) will appear on the next line

   (B) will appear on the previous line

   (C) will appear after having some spaces

   (D) will appear in italic

20) The command in Latex

$\backslash$ [

$\backslash$ left (

$\backslash$ begin {Matrix}

a & b

c & d

$\backslash$ end {Matrix}

$\backslash$ right)

$\backslash$]

generates

(A) $\begin{vmatrix} a & b \\ c & d \end{vmatrix}$
(B) $\begin{Vmatrix} a & b \\ c & d \end{Vmatrix}$

(C) $\begin{Bmatrix} a & b \\ c & d \end{Bmatrix}$
(D) $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$

21) For the largest type size in Latex, the command is

(A) $\backslash$ large
(B) $\backslash$ LARGE

(C) $\backslash$ Huge
(D) $\backslash$ HUGE

22) In Latex, we can make the pages start with any number we want by the command

(A) $\backslash$ page number {page} {number}

(B) $\backslash$ setcounter {page} {number}

(C) $\backslash$ page numbering {page} {number}

(D) $\backslash$ set pagenumber {page} {number}

23) In Scilab, files having the .sce extension contains _____.

    (A) both Scilab functions and executable statements

    (B) neither Scilab functions nor executable statements

    (C) only Scilab functions

    (D) only executable statements

24) In Scilab which of the following statement is correct?

    (A) If we set the variable z to $2 - 3i$, then the output of z'' is $2 + 3i$

    (B) Upper and lower case letters are considered to be same

    (C) The output of *ones* (3, 5) is 3×5 matrix having all entries zero

    (D) The output of *eye* (4, 4) is 4×4 identity matrix

25) A Complete Bipartite graph $K_{n,n}$ has _____ number of edges

    (A) $\dfrac{n(n+1)}{2}$

    (B) n

    (C) n^2

    (D) $\dfrac{n^2}{2}$

26) Let T be a tree with n number of vertices then T has precisely _____ edges

    (A) n

    (B) $n - 1$

    (C) $n + 1$

    (D) n^2

27) Number of composition series of Z are _____

(A) 0

(B) 1

(C) 2

(D) 3

28) Let X be a G-set. Then G_x is _____

(A) subset but not Subgroup of G

(B) subgroup of group G for each x

(C) subgroup of group G for some x

(D) not subset of G

29) Prime field is isomorphic to _____

(A) Q

(B) R

(C) C

(D) set of irrational numbers

30) If F is a finite field of char.F = 3 then number of elements in F are _____

(A) 0

(B) 1

(C) 2

(D) 3

31) The Lagrangian of a particle of mass m in spherical polar co-ordinates is given by $L = \frac{1}{2} m \left(\dot{r}^2 + r^2 \dot{\theta}^2 + r^2 \sin^2 \theta \, \dot{\varphi}^2 \right) - mgl \cos \theta$. The quantity that is conserved is

(A) $\dfrac{\partial L}{\partial \dot{r}}$

(B) $\dfrac{\partial L}{\partial \dot{\theta}}$

(C) $\dfrac{\partial L}{\partial \dot{\varphi}}$

(D) $\dfrac{\partial L}{\partial r}$

32) The extremal of the functional $I = \int_{x_1}^{x_2} x \sqrt{1 + y'^2} \, dx$ is a

(A) catenary

(B) cycloid

(C) arc of the great circle

(D) circle

33) A linear multistep method is convergent if and only if

(A) it is stable and consistent

(B) it is stable

(C) it is consistent

(D) it is bounded

34) To diagonalize the matrix

$$\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$$

By Jacobi's method for symmetric matrices S =

(A) $\begin{bmatrix} \dfrac{1}{\sqrt{2}} & -\dfrac{1}{\sqrt{2}} \\ \dfrac{1}{\sqrt{2}} & \dfrac{1}{\sqrt{2}} \end{bmatrix}$

(B) $\begin{bmatrix} 3 & 0 \\ 0 & -1 \end{bmatrix}$

(C) $\begin{bmatrix} -1 & 0 \\ 0 & 3 \end{bmatrix}$

(D) $\begin{bmatrix} \dfrac{1}{\sqrt{2}} & \dfrac{1}{\sqrt{2}} \\ \dfrac{1}{\sqrt{2}} & \dfrac{1}{\sqrt{2}} \end{bmatrix}$

35) By Charpits method, the auxiliary equations for the partial differential equation

$xpq + yq^2 - 1 = 0$ are

Select one :

(A) $\dfrac{dx}{xq} = \dfrac{dy}{xp + 2yq} = \dfrac{dz}{2(xpq + yq^2)} = -\dfrac{dp}{pq} = -\dfrac{dq}{q^2}$

(B) $\dfrac{dx}{y^2 p + x} = \dfrac{dy}{x^2 + p} = \dfrac{dz}{xp + yq} = \dfrac{dp}{p + q} = \dfrac{dq}{-q^2}$

(C) $\dfrac{dx}{yp} = \dfrac{dy}{xq} = \dfrac{dz}{x^2 p + y^2 q} = \dfrac{dp}{p^2} = \dfrac{dq}{q^2}$

(D) $\dfrac{dx}{x^2 p} = \dfrac{dy}{xp + y^2 q} = \dfrac{dz}{xp + yq^2} = \dfrac{dp}{p} = \dfrac{dq}{-q}$

36) The integral surface of the linear partial differential equation $x(y^2+z)p-y(x^2+z)q=(x^2-y^2)z$ which contains the straight line $x+y=0, z=1$ is Select one :

(A) $x^2+y^2-2z+2=0$

(B) $2xyz+x^2+y^2=0$

(C) $2xyz+x^2+y^2-2z+2=0$

(D) $x^2-y^2-z^2+2xyz=0$

37) Consider the statements :

I) Two solutions $\phi_1(x)$ and $\phi_2(x)$ of L $(y)=0$ are linearly independent on I.

II) $W(\phi_1,\phi_2)(x)\neq 0$ for all $x\in I$. Then

(A) Only I) $\Rightarrow$ II)

(B) I) $\Leftrightarrow$ II)

(C) Only II) $\Rightarrow$ I)

(D) Neither I) $\Rightarrow$ II) nor II) $\Rightarrow$ I)

38) Which of the following is **not** the solution of $y^{(4)}-16y=0$?

(A) $\phi(x)=1$

(B) $\phi(x)=e^{2x}$

(C) $\phi(x)=e^{-2x}$

(D) $\phi(x)=\cos 2x+\sin 2x$

39) Consider the following statements :

(a) The directional derivative $f'(c;u)$ exists if $f'(c)(u)$ exists.

(b) The directional derivative $f'(c;u)$ exists if all partial derivatives exist.
Then _____

(A) (b) is true and (a) is false

(B) Both (a) and (b) are false

(C) Both (a) and (b) are true

(D) (a) is true and (b) is false

40) The line integral of $f(x,y) = y\hat{i} + x^3\hat{j}$ from (0, 0) to (1, 1) along the path $\alpha(t) = t\hat{i} + t\hat{j}$ is equal to _____

(A) 1/2

(B) 0

(C) 3/2

(D) ¾

41) If $T:R^3 \to R^3$ is a linear transformation defined by $T(x,y,z) = (x,y,0)$ then T is _____

(A) one-one

(B) onto

(C) one-one and onto

(D) neither one-one nor onto

42) If a linear transformation $T:R^3 \rightarrow R^3$ is defined by

   $T(x,y,z)=(x+y,y+z,x+y+z)$, then $T^{-1}(a,b,c)$ = _____

   (A) $(b+c,a+b-c,-a+c)$

   (B) $(-b+c,a+b-c,-a+c)$

   (C) $(-b+c,a+b+c,-a+c)$

   (D) $(b+c,a+b-c,a+c)$

43) The function $f(z)=\dfrac{\sin z}{z^3}$ has a

   (A) pole at 1

   (B) pole at 0

   (C) pole at i

   (D) pole at $-i$

44) The radius of convergence of $f(z)=\dfrac{1}{(1-z)}$ is

   (A) 1

   (B) 0

   (C) infinity

   (D) does not exist

45) In the order topology on [0, 1] × [0, 1], closure of the set $A=\{(1/n)\times 0:n\in Z\}$ is

(A) A

(B) $A\cup\{0\}$

(C) 0×0

(D) $A\cup\{(0,0)\}$

46) If $A=\{1/n:n\in Z_+\}$ then closure of A is

(A) $A\cup\{0\}$

(B) A

(C) $\{0\}$

(D) $[0,1)$

47) $\displaystyle\bigcup_{n=1}^{\infty}\left[a+\frac{1}{n},b-\frac{1}{n}\right]=$

(A) $(a, b]$

(B) $[a, b)$

(C) (a, b)

(D) $[a, b]$

48) For $A \subseteq R$ consider the statement

(I) $m^*(A)=0 \Rightarrow A$ is countable

(II) A is countable $\Rightarrow m^*(A)=0$

Then

(A) Only (I) is true

(B) Only (II) is true

(C) Both (I) and (II) are true

(D) Both (I) and (II) are false

49) If x and y are any two vectors in an inner product space then

$$\|x+y\|^2 - \|x-y\|^2 + i\|x+iy\|^2 - i\|x-iy\|^2 =$$

(A) $\langle x, y \rangle$ (B) $2 \langle x, y \rangle$

(C) $3 \langle x, y \rangle$ (D) $4 \langle x, y \rangle$

50) Let T is an operator on Hilbert space H then consider

(I) T*T = I

(II) $\|Tx\| = \|x\|$, for all $x \in H$

Then

(A) (I) $\Rightarrow$ (II) and (II) $\Rightarrow$ (I)

(B) (I) $\not\Rightarrow$ (II) and (II) $\Rightarrow$ (I)

(C) (II) $\not\Rightarrow$ (I) and (I) $\Rightarrow$ (II)

(D) (II) $\not\Rightarrow$ (I) and (I) $\not\Rightarrow$ (II)

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Rough Work