Seat No. Total No. of Pages : 12

Total Marks : 100

M.Phil. / Ph.D. Entrance Examination, October - 2021 MATHEMATICS

Day and Date : Thursday, 21 - 10 - 2021 Time : 01.00 p.m. to 03.00 p.m.

- <u>Instructions</u>: 1) All questions are compulsory.
 - 2) Each question carries 2 marks.
 - 3) Answers should be marked in the given OMR answer sheet by darkening the appropriate option.
 - 4) Use black ball point pen only for marking the circle. Do not make any stray mark on the OMR Answer Sheet.
 - 5) Follow the instructions given on OMR Sheet.
 - 6) Rough work shall be done on the sheet provided at the end of question paper.
 - 7) Only non-programmable calculators are allowed.

Choose the correct answer :

- 1) Research ethics do not include
 - A) Honesty B) Subjectivity
 - C) Integrity D) Objectivity
- 2) The meaning of variable is
 - A) The quality having different values
 - B) A quantity having different values
 - C) Both of the above
 - D) None of the above
- **3**) Conferences are meant for
 - A) Multiple target groups B) Group discussions
 - C) Showcasing new research
- D) All the above

- 4) Ex post facto research means the research is carried out
 - A) after the incident
 - B) prior to the incident
 - C) along with the happening of the incident
 - D) keeping in mind the possibilities of an incident
- 5) The research is a
 - A) Honest investigation
 - B) Passive investigation
 - C) Investigation for showing scholarship
 - D) Investigation in the interest of society
- 6) The experimental method is
 - A) A method for verifying a hypothesis
 - B) A method deriving inferences
 - C) A variable controlling method
 - D) None of the above
- 7) The research that aims at immediate application is
 - A) action research B) empirical research
 - C) conceptual research D) fundamental research
- 8) The characteristic feature of a workshop is
 - A) To attain the higher cognitive a objectives of the research
 - B) To summarize the theoretical and practical aspects of the research
 - C) To explore the possibilities of applied aspects of the research
 - D) All of the above
- 9) The advantages of preparing the research paper is
 - A) The exchange of ideas related to research
 - B) The familiarity with the research approaches
 - C) The awareness about present researches
 - D) All of the above

- 10) What will you do to make your research value oriented?
 - A) You pay the cost of its writing to a ghost writer
 - B) You will charge the market rate for the work, you have conducted so far
 - C) You will ensure honesty and faith in the research work
 - D) You will make a Plagiarism
- **11**) The research paper and the research article are
 - A) Two separate name of one and same thing
 - B) First is factual and later a theoretical in nature
 - C) First is having survey background the latter is experiment in nature
 - D) b and c are the correct statements
- **12**) The meaning of generalization is
 - A) To normalize special quality
 - B) To implement the research conclusion at the larger level
 - C) To give advantages of research to normal person
 - D) None of the above
- 13) Which of the following is the correct mathematical writing
 - A) $a_1, a_2, ..., a_n$ B) $a_1, a_2 ... a_n$
 - C) $a_1, a_2, \dots a_n$ D) $a_1, a_2, \dots a_n$

14) Which of the following is an example of professional writing?

- A) Fermats Theorem B) Fermat's theorem
- C) Fermat's Theorem D) Fermat theorem

- 15) Which of the following is an example of professional writing?
 - A) Dirichlet's function $f: \rightarrow$ is defined as f(x) = 0, if x is a rational number and f(x) = 1, if x is an irrational number.
 - B) Dirichlet Function $f: \rightarrow$ is defined as f(x) = 0, if x is a rational number and f(x) = 1, if x is an irrational number.
 - C) Dirichlet function $f: \rightarrow$ is defined as f(x) = 0, if x is a rational number and f(x) = 1, if x is an irrational number.
 - D) Dirichlets function $f: \rightarrow$ is defined as f(x) = 0, if x is a rational number and f(x) = 1, if x is an irrational number.
- 16) Which of the following symbols are permitted in professional writing?
 - A) \therefore but not \therefore B) Both \therefore and \therefore
 - C) neither \therefore and \therefore D) only \therefore
- 17) Which of the following is an example of professional writing?
 - A) State and prove Open Mapping theorem
 - B) State and prove Open mapping theorem
 - C) State and prove open mapping theorem
 - D) State and Prove Open Mapping Theorem
- **18**) Which of the following is an example of professional writing?
 - A) I prove the Theorem 6 on page 16
 - B) We prove the Theorem 6 on Page 16
 - C) I prove the Theorem 6 on Page 16
 - D) We prove the Theorem 6 on page 16
- **19**) The default type that Tex produces is of
 - A) 10 pt size B) 11 pt size
 - C) 12 pt size D) 14 pt size

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20) The Latex command $\langle equation^* \rangle$ $x \mid |e |x| \mid |u | |x| \mid |$

21) The equation $(x^m)^n = x^{mn}$ produced by typing

A)	$x_m^n = {x^m}^n $	B)	$(x^m)^n = x^{mn}$
C)	$ \{x^{m}\}^{n} = x^{m} $	D)	$ [x^m]^n = x^[mn] $

22) In Latex $\hat{f}(z)dz$ produces

A)
$$\oint f(z)dz$$
B) $\int f(z)dz$ C) $\oint f(z)dz$ D) $\iint f(z)dz$

 23) The output of the Scilab command 5:2:9 is

 A) 5 2 9
 B) 5 7 9

 C) 5 6 7
 D) 7 8 9

24) The statement a <> b is true in Scilab if _____ A) a = b B) a > b

C) a < b D) $a \neq b$

25) A complete graph K_n has _____ number of edges.

A)
$$\frac{n(n+1)}{2}$$
 B) $(n+1)$

C)
$$\frac{n(n-1)}{2}$$
 D) $\frac{(n+1)}{2}$

- 26) Adjacency matrix is _____
 - A) symmetric B) antisymmetric
 - C) identity D) scalar

27) Let G = (Z, +, .) and H = (3Z, +, .) then number of distinct left cosets of H in G are _____

 A) 0
 B) 1

 C) 2
 D) 3

28) For a commutator subgroup G', which of the following is not true?

A)	G' is normal in G	B)	$\frac{G}{G'}$ is abelian
C)	G' is not normal in G	D)	G' is subgroup of G

- 29) For an algebraic closure E of F which of the following is not true?
 - A) E is an algebraic extension
 - B) E is algebraically closed
 - C) E is not algebraically closed
 - D) E contains all roots of polynomial over F

30) If K is a splitting field of $f(x) \in F[x]$ then K is _____

- A) Finite Extension
- B) Infinite Extension
- C) Finite but not Algebraic Extension
- D) Algebraic but not finite extension

31) Suppose
$$H = \frac{p^2}{2} - \frac{q^2}{2}$$
 with $q(0) = p(0) = 1$.

- A) *p* and *q* increases as *t* increases
- B) p increases and q decreases as t increases
- C) p and q decrease as t increases
- D) q increases and p decreases as t increases

32) The extremals of the functional $\int_0^1 (y'^2 + x^2) dx$ subject to conditions y(0)=0, y(1)=0 is

A)
$$y = x$$
 B) $y = 0$

C)
$$x = 0$$
 D) $y = -x$

33) Jacobi's method for symmetric matrices is used to calculate

- A) only eigenvalues of the matrix
- B) eigen vectors of the matrix
- C) eigen values and eigen vectors
- D) eigen values and characteristic polynomial

34) Order conditions of second order Runge Kutta method are,

A)
$$\sum_{i=1}^{2} b_i = 1, b_2 c_2 = \frac{1}{2}$$

B) $\sum_{i=0}^{2} b_i = 1, b_2 c_2 = \frac{1}{2}$
C) $\sum_{i=1}^{2} b_i = 1, \sum_{i=1}^{2} b_i c_i = \frac{1}{2}$
D) $\sum_{i=1}^{2} b_i = \frac{1}{2}, b_2 c_2 = 1$

35) By Jacobis method, the auxiliary equations for the partial differential equation $z^2 + zu_z - u_x^2 - u_y^2 = 0$ are select one :

A)
$$-\frac{dx}{2u_x} = -\frac{dy}{2u_y} = \frac{dz}{z} = \frac{du_x}{0} = \frac{du_y}{0} = \frac{du_z}{-2z - u_z}$$

B)
$$\frac{dx}{u_x^2} = \frac{dy}{u_y^2} = \frac{dz}{z^2} = \frac{du_x}{0} = \frac{du_y}{0} = -\frac{du_z}{u_z^2}$$

C)
$$\frac{dx}{xu_x} = \frac{dy}{yu_y} = \frac{dz}{zu_z} = \frac{du_x}{u_x^2} = \frac{du_y}{u_y^2} = \frac{du_z}{u_z^2}$$

D)
$$\frac{dx}{u_x} = \frac{dy}{u_y} = \frac{dz}{z} = \frac{du_x}{0} = \frac{du_y}{0} = -\frac{du_z}{u_z}$$

36) For $\xi = y + \frac{x^2}{2}$ and $\eta = y - \frac{x^2}{2}$, the equation $u_{xx} - x^2 u_{yy} = 0$ reduces to the canonical form select one :

A)
$$u_{\zeta\eta} + u_{\zeta} + u_{\eta} = 0$$

B) $u_{\zeta\eta} = 0$
C) $u_{\xi\eta} = \frac{1}{4(\xi - \eta)}(u_{\xi} - u_{\eta})$
D) $u_{\zeta\eta} + u_{\eta\zeta} = 0$

37) Which of the following is the solution of initial value problem given by y'' - 4y = 0, y(0) = 0, y'(0) = 1?

A)
$$\frac{1}{4}(e^{2x} - e^{-2x})$$

B) $\frac{1}{4}(e^{-2x} - e^{2x})$
C) $\frac{1}{4}(e^{2x} + e^{-2x})$
D) $e^{-2x} + e^{2x}$

- **38)** Consider the statements
 - i) If ϕ_1 and ϕ_2 are linearly independent functions on an interval I, then they are linearly independent on any interval $J \subset I$.
 - ii) If ϕ_1 and ϕ_2 are linearly independent solutions of L(y) = 0 on an interval I, then they are linearly independent on any interval $J \subset I$. Then
 - A) Both i) and ii) are false B) i) is true but ii) is false
 - C) Both i) and ii) are true D) i) is false but ii) is true
- **39**) A linear operator A on a finite dimensional vector space X is _____ if and only if it is onto.
 - A) bounded B) linear
 - C) one-to-one D) continuous

40) If f(x) = 3x is a linear function then f'(c; u) =

- A) f(u) B) f(C)
- C) f(C) + f(u) D) f(C)f(u)

41) If $T:V \rightarrow W$ and $S:W \rightarrow U$ be two linear transformations such that ST is one-one and onto

then _____

- A) T is onto
- B) S is one one
- C) T is one-one and S is onto
- D) T is not one-one and S is not onto

42) If $T:V_2 \rightarrow V_3$ is a linear transformation defined by $T((x_1, x_2) = (x_1 - x_2, x_2 - x_1, -x_1))$ then nullity of T =______ A) 0 B) 1

C) 2 D) 3

43) "Every bounded entire function is constant" is the statement of

- A) Cauchy's theoremB) Morera's theorem
- C) Liouville's theorem D) Leibnitz theorem

44) Let $G = \{z \in \not\subset : |z - i| < \sqrt{3}\}$ and $f : G \to \not\subset$ a differentiable function. Then analyticity of *f* follows from

- A) Cauchy's theorem B) Goursat's theorem
- C) Morera's theorem D) None of the above
- **45**) Which one of the following is always true?
 - A) Continuous image of compact space is compact.
 - B) A compact subset of Hausdorff space is not closed
 - C) The closed interval [1, 5] is not compact in R^1
 - D) The set $K = \{0\} \cup \left\{\frac{1}{n} : n \in Z^+\right\}$ is not compact in \mathbb{R}^1
- **46**) Which of the following functions from R to R^{ω} are continuous in both box and product topologies on R^{ω} ?

A)
$$f(t) = (t, 2t, 3t,)$$

B) $f(t) = (t, t, t,)$
C) $f(t) = (t, t^2, t^3,)$
D) $f(t) = \left(t, \frac{1}{2}, \frac{1}{3}, ...\right)$

- **47)** If N is a normed linear space and x_0 is non zero vector in N, then always there is a functional $f_0 \in N^*$ such that
 - A) $f_0(x_0) = ||f_0||$ B) $f_0(x_0) = x_0$ C) $||f_0|| = ||x_0||$ D) $f_0(x_0) = ||x_0||$

48) If M is closed linear subspace of a Hilbert space H then always

- A) $M^{\perp} \cap M = \{0\}$ but $M + M^{\perp} \neq H$
- B) $M^{\perp} \bigcap M \neq \{0\}$ and $M + M^{\perp} = H$
- C) $M^{\perp} \bigcap M \neq \{0\}$ and $M + M^{\perp} \neq H$
- D) $M^{\perp} \cap M = \{0\}$ and $H = M + M^{\perp}$

49) Define $f: [0, 1] \rightarrow \mathbb{R}$ by

 $f(x) = \begin{cases} 1 & \text{if } x \text{ is rational} \\ 0 & \text{if } x \text{ is irrational} \end{cases}$

Then,

$$\int f = [0, 1]$$
A) 0 B) 1
C) ∞ D) does not exist

50) Let E = (0, 1]. For each n (n = 1, 2, 3,) define $f_n = n \chi_{[0,\frac{1}{n}]}$ on E

Then
$$\lim_{n \to \infty} \int_{E} f_n =$$

A) 0 B) 1
C) 2 D) ∞

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Rough Work

Rough Work